International Conference
on Partial Differential Equations and Applications
in Memory of Professor B. Yu. Sternin
Moscow, Russia, November 6–9, 2018

ABSTRACTS

RUDN University,
Moscow State University,
Ishlinsky Institute for Problems in Mechanics RAS

Moscow
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Конференция посвящена памяти профессора Бориса Юрьевича Стернина (1939–2017). Научная программа включает доклады по разным разделам теории дифференциальных уравнений с частными производными и их приложениям, которыми занимался проф. Б.Ю. Стердин. В работе конференции принимают участие ведущие российские и зарубежные специалисты, а также его коллеги, ученики и соавторы.
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Extension of an orthogonally additive map dominated by a continuous operator

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Today, the theory of orthogonally additive operators in vector lattices is an active area of functional analysis (see, for instance, [1, 2, 6, 7, 8])). In the recent years some connections with the problems on convex geometry emerged [5, 9].

Let $E$ and $F$ be vector lattices.

Recently, in the forthcoming paper [3] it was proved that a dominated orthogonally additive operator $T: V \rightarrow W$ is laterally-to-order continuous if and only if so is its exact dominant $|T|: E \rightarrow F$. An element $y$ of a lattice-normed space $V$ is said to be a fragment of an element $x \in V$ (in another terminology, a component) if $y \perp (x - y)$. The set of all fragments of $x$ is denoted by $F_x$. A subset $D$ of a lattice-normed space $V$ is called a lateral ideal if the following conditions hold:

- if $x \in D$, then $y \in D$ for any $y \in F_x$;
- if $x, y \in D$, $x \perp y$, then $x + y \in D$.

**Theorem.** Let $(V, E)$ be a lattice-normed space, let $(W, F)$ be a Banach–Kantorovich space over a Dedekind complete vector lattice $F$, let $D$ be a lateral ideal in $V$, and let $T: D \rightarrow W$ be an orthogonally additive map dominated by a laterally-to-order continuous (σ-laterally-to-order continuous) positive orthogonally additive operator $S: E \rightarrow F$. Then there exists a dominated laterally-to-order continuous (σ-laterally-to-order continuous) orthogonally additive operator $\tilde{T}_D: V \rightarrow W$ such that $\tilde{T}_D x = T x$ for any $x \in D$.

**References**

A remark on the Mahowald elements
and on iterated desuspension in the stable homotopy groups of spheres

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A new geometrical approach toward a description of $E_2$ term of the Adams spectral sequence is presented. This approach is based on skew-framed immersion cobordism group [1]. As an example, a geometrical description of the Mahowald elements [2] of dimension $2^i$, $i \geq 3$ is presented. The approach allows to compare the desuspension problem for stable homotopy groups $\Pi_{128}$ and $\Pi_{126}$. The Mahowald element in $\Pi_{128}$ admits no 5-time desuspensions into the unstable range. Barratt–Jones–Mahowald approach relates the Snaith Conjecture [3] about Arf-invariant in dimension 126 with the desuspension problem.

This is a part of joint results with Th. Yu. Popelenskii and O. D. Frolkina.

References

Cohomology of n-categories and derivations
in group algebras

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This work represents the concept of an n-groupoid $\Gamma^n$ and n-characters $\chi_n$ on n-groupoids as complex-valued maps from spaces of different classes of morphisms satisfying the condition $\chi_n(\psi \circ_\kappa \varphi) = \chi_n(\psi) + \chi_n(\varphi)$ for any possible compositions. A sequence of spaces of n-characters and morphisms between them is constructed and its exactness is shown. This construction has an important application for describing derivations in group algebras. In particular, this approach allows us to study the outer derivation algebra from a new point of view and also construct some interesting examples. The research was supervised by Professor Arutyunov and is based on ideas due to Professor Mishchenko.

References
Regularity of a boundary point for the $p(x)$-Laplacian

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In a bounded domain $D \subset \mathbb{R}^n$, $n \geq 2$ consider the generalized Dirichlet problem

$$Lu_f = \text{div} \left( |\nabla u_f|^{p(x)-2} \nabla u_f \right) = 0 \quad \text{in} \quad D, \quad u_f|_{\partial D} = f \in C(\partial D).$$

with a measurable exponent $p$ satisfying $1 < \alpha \leq p(\cdot) \leq \beta < \infty$ a.e. in $D$. A solution corresponding to a continuous boundary function is obtained as a limit of variational solutions with smooth boundary data, see [1] for details. Due to the Lavrentiev phenomenon two different types of solutions may arise — $H$- and $W$-solutions [2]. Thus the following definition can be considered in the $H$- and $W$-frameworks.

**Definition 1.** The boundary point $x_0 \in \partial D$ is called regular if $\text{ess lim}_{D_{2t} \ni x \to x_0} u_f(x) = f(x_0)$ for any function $f \in C(\partial D)$.

We assume that the exponent $p$ is essentially continuous at the boundary point $x_0 \in \partial D$, and $\text{ess osc}_D \{ p, B_{2r}^{x_0} \} \leq \omega(r)$, where $B_{2r}^{x_0}$ is an open ball of radius $r$ centered at $x_0$, and $\omega$ is a continuous, nondecreasing on $(0, d]$ function, such that $\omega(0) = 0$. We set $p(x) = p(x_0)$ for $x \in \mathbb{R}^n \setminus D$.

**Definition 2.** (see [1]). The $H$-capacity ($W$-capacity) of a compact set $K \subset B_{2r}^{x_0}$ with respect to the ball $B_{2r}^{x_0}$ is the number $C_p(K, B_{2r}^{x_0}) = \inf_{B_{2r}^{x_0}} \int_{B_{2r}^{x_0}} |\nabla \varphi|^{p(x)}(p(x))^{-1} \, dx$, where the infimum is taken over the set of function $\varphi \in C_0^\infty(B_{2r}^{x_0})$ ($\varphi \in W_0^1(B_{2r}^{x_0})$) that are greater than or equal to 1 almost everywhere on $K$. For $H$- and $W$-solutions we use $H$- and $W$-capacities, correspondingly.

Let the function $\theta(r) = r^{-\omega(r)}$ be nonincreasing on $(0, d)$. We denote $p_0 = p(x_0)$.

**Theorem 3.** Let

$$\int_0^\infty \exp \left( -c_i \theta^{\gamma_i}(t) \right) \left( C_p \left( B_{2t}^{x_0} \setminus D, B_{2t}^{x_0} \right)^{p_0-n} \right)^{1/(p_0-1)} t^{-1} dt = \infty,$$

where $c_i = c_i(n, \alpha, \beta) > 0$, $i = 1, 2$. Then the boundary point $x_0 \in \partial D$ is regular.

For $p = \text{const}$ this theorem was obtained in [3], and for $\omega(t) = C/(\ln 1/t)^{-1}$ in [1].

This work was supported by the Ministry of Education and Science of the Russian Federation (project No. 1.3270.2017/4.6).

References

Parametrix and asymptotics of rapidly varying solutions for linearized equations of gas dynamics

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We study system of equations of gas dynamics with small viscosity linearized on a smooth external flow. The parabolic part of the system is degenerate; for the vanishing viscosity, we obtain nonstrictly hyperbolic system (containing simple as well as multiple characteristics). For this system, we describe the expansion of the resolving operator, which is asymptotic with respect to smoothness and to small viscosity simultaneously. The asymptotic series consists of summands of two different types: they describe acoustic and hydrodynamic modes. For hydrodynamic modes, there are no focal points (characteristics lie in configuration space); the amplitude is computed from the system of ODE’s along the characteristics. For the acoustic modes, we use Maslov canonic operator to describe the summands of asymptotic series. The amplitudes (functions on the corresponding Lagrangian manifolds) can be computed explicitly. We apply this construction to the description of short-wave asymptotics. We show that for certain external flows the hydrodynamic modes can grow in time.

This work was supported by RFBR (grants 18-31-00273, 17-51-150006).

A trace formula for foliated flows

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Let \( \mathcal{F} \) be a smooth codimension one foliation on a compact manifold \( M \). A flow \( \phi^t \) on \( M \) is said to be foliated if it maps leaves to leaves. If moreover the closed orbits and preserved leaves are simple, then there are finitely many preserved leaves, which are compact, forming a compact subset \( M^0 \), and a precise description of the transverse structure of \( \mathcal{F} \) can be given. A version of the reduced leafwise cohomology,
\( \Pi I(\mathcal{F}) \), is defined by using distributional leafwise differential forms conormal to \( M^0 \).
The talk will be about our progress to define distributional traces of the induced action of \( \phi^t \) on \( \Pi I(\mathcal{F}) \), for every degree \( r \), and to prove a corresponding Lefschetz trace formula involving the closed orbits and leaves preserved by \( \phi^t \). The formula also involves a version of the \( \eta \)-invariant of \( M^0 \). This kind of distributional trace formula was conjectured by Christopher Deninger [3], and it was proved by the first two authors when \( M^0 = \emptyset \) [1, 2].

References

A Baum–Connes conjecture for singular foliations and its use

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In several cases, a non-compact manifold can be understood as a dense leaf of a singular foliation. We will discuss how, in cases as such, spectral gaps of the Laplacian may be detected using the \( K \)-theory of the foliation \( C^* \)-algebra. The “shape” of this \( K \)-theory is described by a Baum–Connes assembly map for singular foliations.

References
Asymptotic solutions of stationary problems for linearized plasma equations

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We propose a method described in [1] for calculating asymptotic solutions of stationary problems for differential (or pseudodifferential) operators whose symbol is a self-adjoint matrix. We show that the problem of constructing asymptotic solutions corresponding to a fixed eigenvalue (called an effective Hamiltonian, term, or mode) reduces to studying objects related only to the determinant of the principal matrix symbol and the eigenvector corresponding to a given (numerical) value of this effective Hamiltonian. We apply this method in a linearized $12 \times 12$ system describing plasma motion (e.g. in tokamak).

This work was supported by the RFBR grant 18-31-00273.

References

C*-algebras generated by dynamical systems, and applications

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The main problems we are dealing with in the present report originate in the study of the operators acting in some function spaces over a set $X$ of the form

$$Bu := \sum_k a_k u(\alpha_k(x)) = f(x), \quad x \in X, \quad (1)$$

where $\alpha_k : X \to X$ are certain transformations of the domain $X$ and $a_k$ are operators from some operator’s algebra $A$, as a rule well-investigated. If $X$ is a manifold and $a_k = a_k(x,D)$ are pseudodifferential operators (PDO) on $X$, then the operator (1) is a nonlocal pseudodifferential operator. If $a_k = a_k(x)$ are operators of multiplication by functions or matrix functions, then the operator (1) is a “pure” functional operator. An operator of the form $bu = a(x)u(\alpha(x))$ is called a weighted shift operator.
The report contains a short survey of the results on this subject given in [1, 2], as soon as some new results from [1, 2].

We present the following results:
1. General approach to investigate these operators based on the theory of $C^*$-algebras generated by dynamical systems.
2. Symbolic calculus for nonlocal pseudodifferential operators. The symbol $\sigma(B)$ of such operator $B$ is a “pure” functional operator acting in a function space on the cotangent bundle of the manifold $X$.

**Theorem.** A nonlocal pseudodifferential operator $B$ is Fredholm if and only if its symbol $\sigma(B)$ is invertible. The operator $B$ is semi-Fredholm if and only if its symbol $\sigma(B)$ is one-sided invertible.

3. The description of the spectral properties of weighted shift operator and invertibility conditions for a “pure” functional operator.
4. One-sided invertibility conditions for weighted shift operators.

**References**

**Exact solutions of a nonclassical nonlinear partial differential equation**

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We study the equation

$$\frac{\partial^2 u}{\partial t \partial x} + w^p \frac{\partial u}{\partial x} = u^q. \quad (1)$$

Here $u(\cdot)$ is a real function of the space variable $x \in R$ and time $t > 0$. The parameters $p$ and $q$ are integer; moreover, $p \geq 0$, $q \geq 1$, $p + q \geq 2$.

Earlier (see [1]), the similar equation

$$\frac{\partial^2 u}{\partial t \partial x} + \frac{\partial}{\partial x} (|u|^{p-2} u) = |u|^q$$

was studied. It describes a nonstationary process in a semiconductor with regard to heating. Sufficient conditions for the nonexistence of generalized solutions of the Cauchy problem and some other problems for the equation on every finite time interval were obtained.
Our aim is to construct exact solutions of (1) and to analyse their qualitative properties. Fourteen classes of exact solutions of (1) are constructed in the present work. Among them are both bounded solutions and solutions going to infinity when time goes to a finite value.

We use separation of variables and the method of differential constraints. Also, travelling wave solutions, self-similarity solutions, and some special solutions are constructed. Some computations are automated with the Maple system.

This work was supported by the Presidential Program for Support of Young Philosophy Doctors (project no. MK-1829.2018.1).

References

Derivations on group algebras

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The main theme of the report is a combinatorial description of derivations on a group algebra. The linear mapping \( d : \mathbb{C}[G] \to \mathbb{C}[G] \) is said to be the derivation if the Leibniz rule holds

\[
    d(uv) = d(u)v + ud(v), \forall u, v \in \mathbb{C}[G].
\]

We can consider a groupoid \( \Gamma \) in the following way.

The set of objects is a set of elements \( \text{Obj}(\Gamma) = \{ g \in G \} \). The set of arrows is a set of pairs \( \text{Hom}(\Gamma) := \{ (u, v) \mid u, v \in G \} \). An arrow \( \phi = (u, v) \) has a source \( s(\phi) = v^{-1}u \) and a target \( t(\phi) = uv^{-1} \). For arrows \( \phi = (u_1, v_1) \) and \( \psi = (u_2, v_2) \) such that \( t(\phi) = s(\psi) \) we will define a composition

\[
    \psi \circ \phi := (v_2u_1, v_2v_1).
\]

We will call a character any mapping \( \chi : \text{Hom}(\Gamma) \to \mathbb{C} \) such that \( \chi(\psi \circ \phi) = \chi(\psi) + \chi(\phi) \) when \( t(\phi) = s(\psi) \). We will call a character a locally finite character if \( \forall v \in G\chi((u, v)) = 0 \), for all except of finite number of elements \( u \in G \).

Theorem 1. For any derivation \( d \) exist a character \( \chi_d \) such that \( \forall a \in G \)

\[
    d(a) = a \left( \sum_{t \in [a]} \chi_d(at, a)t \right).
\]

This theorem gives a possibility to calculate an algebra of derivations for different group algebras. Let \( G \) be a central commutative extension group. Then we get a theorem. Let \( \tau : G \to \mathbb{C} \) be a homomorphism on an additive group of complex numbers. The space of such homomorphisms we will denote by \( T(G) \). And let \( z \) be an element of a group centre \( z \in Z(G) \). Then

\[
    d^z_\tau(g) = \tau(g)gz.
\]
Let $G^+$ be a set of elements such that $G/Z(u)$ has a nonfinite cyclic subgroup, but doesn’t have a subgroup isomorphic to a a free abelian group of rank 2. Then
\[ d_u(x) = z(t^{-1}u + t^{-2}u^2 + \cdots + t^{-l}u^l). \]

**Theorem 2.** Any derivation $d$ can be written in the following way
\[ d = a_z \tau_i d_{\tau_i} z_j + b_l d_{u_i} + \ldots \]
where $d_{\tau_i} z_j, z_g \in Z(G)$, $\tau_i$ are generators of $T(G)$, and $d_{u_i}$ for $u_i \in G^+$. By \ldots we mean inner derivations. Sure, almost all coefficients $a_z, b_l$ have to be equal to 0.

**References**

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**On atypicality of power-law solutions to highly superlinear Emden–Fowler type equations**

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Consider the equation
\[ y^{(n)} = p(x, y, y', \ldots, y^{(n-1)} | y|^k \text{ sign } y, \quad n \geq 2, \quad k > 1. \] (1)

Results on asymptotic behavior of blow-up solutions to this equation are presented. In particular, for slightly superlinear equations we prove Kiguradze’s hypothesis (see [1], Problem 16.4) about power-law behavior of all blow-up solutions.

**Theorem 1** ([2]). Suppose $p \in C(\mathbb{R}^{n+1}) \cap Lip_{y_0, \ldots, y_{n-1}}(\mathbb{R}^n)$ and $p \to p_0 > 0$ as $x \to x^*, y_0 \to \infty, \ldots, y_{n-1} \to \infty$. Then for any integer $n > 4$ there exists $K > 1$ such that for any real $k \in (1, K)$, any solution to equation (1) tending to $+\infty$ as $x \to x^*-0$ has power-law asymptotic behavior, namely
\[ y(x) \sim C(x^*-x)^{-\alpha}(1 + o(1)), \quad \alpha = \frac{n}{k-1}, \quad C^{k-1} = \frac{1}{p_0} \prod_{j=0}^{n-1} (j + \alpha). \]

The existence of blow-up solutions with non-power-law behavior was also proved, which means that in general case this hypothesis is not true. (See [3] and [4]).

It appears that the asymptotic behavior of solutions to equation (1) depends on the spectrum of a related linear operator, and in general case, even for equation (1) with $p = p_0 > 0$, the power-law behavior of blow-up solutions is atypical.

**Theorem 2.** Suppose $n \in \mathbb{N}, \alpha \in (0; +\infty)$, and the algebraic equation $\prod_{j=0}^{n-1} (\lambda + j + \alpha)$ has at least two roots with positive real part and no purely imaginary root. Then solutions to the differential equation (1) with $p = p_0 > 0$ having the power-law asymptotic behavior $y(x) \sim C(x^*-x)^{-\beta}$ as $x \to x^*-0$ with any $C \neq 0, \beta > 0, x^* < \infty$ are atypical, i.e. at any point $x_0$ the subset of all initial data producing solutions with the above power-law asymptotic behavior has null Lebesgue measure in $\mathbb{R}^n$. 
Corollary 1. Suppose \( n > 11 \) and the equation \( \prod_{j=0}^{n-1}(\lambda + j) = \prod_{j=0}^{n-1}(1 + j) \) has no purely imaginary root. Then there exists \( K_n > 1 \) such that for any \( k > K_n \), solutions to equation (1) with \( p = p_0 > 0 \) with the power-law asymptotic behavior are atypical.

References

The equivariant Atiyah–Patodi–Singer theorem
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In 1975, Atiyah, Patodi and Singer proved a remarkable index theorem for a first order elliptic operator, \( D \), acting on the smooth sections of a complex vector bundle over a compact manifold. The key point was that the manifold now was supposed to have a boundary, such that elliptic boundary conditions had to be imposed for \( D \). The natural intuition that suitable local boundary conditions would work turned out to be wrong; what the authors brought in were spectral boundary conditions based on the restriction of \( D \) to the boundary, that yielded an elliptic first order operator, \( A \). Then the boundary condition was expressed in terms of the spectral projection \( P_c(A) \). The index formula, therefore, was expected to involve interior as well as boundary contributions. This could be verified and resulted in the local term, integrated over the manifold, and the famous eta-invariant of the operator \( A \).

The purpose of this talk is to present the equivariant version of this result. We assume that a compact Lie group, \( G \), acts on all data. This means that the action of \( G \) is restricted to the (infinite dimensional) isotypical subspace formed by an irreducible representation; this is a subspace of the original space hence not the space of sections of some smooth vector bundle over the manifold. Nevertheless, we can prove a completely analogous index formula for the restricted operator that is independent of the complexity of the Thom–Mather space arising from the \( G \)-orbit spaces of the isotypical subspaces.
Stability estimates for the eigenvalues of higher order elliptic operators upon domain variation

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We consider the eigenvalue problem for the operator

$$Hu = (-1)^m \sum_{|\alpha|=|\beta|=m} D^\alpha (A_{\alpha\beta}(x) D^\beta u), \quad x \in \Omega,$$

subject to homogeneous Dirichlet or Neumann boundary conditions, where $m \in \mathbb{N}$, $\Omega$ is a bounded open set in $\mathbb{R}^N$ and the coefficients $A_{\alpha\beta}$ are real-valued Lipschitz continuous functions satisfying $A_{\alpha\beta} = A_{\beta\alpha}$ and the uniform ellipticity condition

$$\sum_{|\alpha|=|\beta|=m} A_{\alpha\beta}(x) \xi_\alpha \xi_\beta \geq \theta |\xi|^2$$

for all $x \in \Omega$ and all $\xi_\alpha \in \mathbb{R}$, $|\alpha| = m$, where $\theta > 0$ is the ellipticity constant.

We consider open sets $\Omega$ for which the spectrum is discrete and can be represented by means of a non-decreasing sequence of non-negative eigenvalues

$$\lambda_1[\Omega] \leq \lambda_2[\Omega] \leq \cdots \leq \lambda_n[\Omega] \leq \cdots$$

where each eigenvalue is repeated as many times as its multiplicity.

We present estimates for the variation

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]|$$

of the eigenvalues corresponding to two open sets $\Omega_1$, $\Omega_2$. Our analysis comprehends open sets with arbitrarily strong degenerations.

Motion on manifolds with singularities

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The main problem of our research is the construction of some analog of differential calculus for manifolds with singularities. This problem arises in theoretical mechanics when a mechanism may have some branching points where it can change the type of motion. It is important to answer the following question: what is an ODE on manifolds with singularities? Classical techniques for deriving equations of motion such as Newton’s second law and the minimum action principle have difficulties.

We study the case when the model space $X$ consists of two smooth curves $\gamma_1$ and $\gamma_2$ which are tangent at a point $x$. From general considerations we could predict that motion have to be smooth. But is there a possibility for system to change curve after...
passing through the singular point? The following geometric approach is native. We could study $X$ as an object of $\mathcal{S}$, some category of manifolds with singularities. For the case of manifolds $M$ the equation of motion is a vector field on the tangent bundle $TM$. In the case of a space $X \in \mathcal{S}$ we generalize this scheme: it is required that for all $Y \in \mathcal{S}$ there exists a “tangent bundle” $TY \in \mathcal{S}$ and for all $Y \in \mathcal{S}$ the class of “smooth” vector fields $\mathcal{V}_Y$ and the class of “smooth” curves $r: \mathbb{R} \to Y$ are defined.

Calculations in this way when $\mathcal{S}$ is the category of Frölicher spaces [1, 2]) will be shown. The example of mechanical system with singularities could be found in [3].

References

Asymptotics for random walks on metric graphs

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Let us consider a random walk on a finite compact metric graph (see, for example, [1]). Let one point move along the graph at the initial moment of time. The passage time for each individual edge is fixed. In each inner vertex, the point with some probability selects one of the edges for further movement. The reflection occurs in the vertices of valence one. Backward turns on the edges are prohibited. The aim is to analyze the asymptotic behavior of the number $N(T)$ of possible endpoints of such random walk on the metric graph as time $T$ increases. We suppose that the probability of choosing an edge is non-zero for all edges. It is a situation of a general position.

Such random walk is typical for evolution of wave packets, localized in a small neighborhood of one point at the initial moment of time (see [2] and references therein).

In the case of linearly independent over the rationals lengths the problem is related to the problem of counting the number of lattice points in expanding simplices with real vertices. An asymptotic expansion for $N(T)$ using Barnes’ multiple Bernoulli polynomials [5, 7] (also known as Todd polynomials, see [6] for details) was found. Explicit formulas for the first two terms of the expansion for the counting function of the number of moving points are presented (see [4] for details). The leading term was found earlier (see [2]) and depends only on the number of vertices $V$, the number of edges $E$ and the sum and the product of lengths of the edges $t_j$.

The second term of the asymptotics is more complicated. It is determined by the quadratic form (i.e. $E$ by $E$ symmetric matrix) of the lengths of the edges of the metric graph. And it, generally speaking, depends on the starting vertex.

Let $G$ denote a finite connected subgraph of the graph $\Gamma$, containing the starting vertex $s$. For the subgraph $G \subset \Gamma$ the vertex set is denoted by $V(G)$ and the set of edges by $E(G)$. For $v \in V(G)$, we denote by $\rho(G,v)$ the valency of the vertex.
v in the subgraph $G$. The vertex $v$ is said to be the end-vertex in the subgraph $G$ if $\rho(G, v) = 1$. Then the unique edge $e \in E(G)$ is called the end-edge. The edge $e \in E(G)$ is called the isthmus, if after deleting this edge the graph $G$ splits into two connected components.

**Theorem 1.** Suppose that the finite graph $\Gamma$ has edge lengths $t_1, \ldots, t_E$, linearly independent over $\mathbb{Q}$, and the starting vertex $s$ is not an end-vertex, then the counting function has the decomposition $N(T) = N_1 T^{E-1} + N_2 T^{E-2} + o(T^{E-2})$, where

$$N_2 = \frac{1}{2^{E-2}(E-2)!} \prod_{i=1}^{E} t_i \left[ \sum_{j=1}^{E} \sum_{i=1, i \neq j}^{E} t_j t_i \gamma_{i,j} 2^{\beta_1(\Gamma \setminus \{e_i\})} - 2^{\beta_1(\Gamma)} \sum_{e_j} t_j \right]$$

$$+ \sum_{\{e_i, e_j\}} (4 - m) 2^{\beta_1(G)} t_i t_j + \sum_{\{e_i, e_j\}} 2^{\beta_1(G) + \delta_{i,j}} t_i t_j - \sum_{\{e_i, e_j\}} 2^{\beta_1(G)} t_i t_j \right].$$

Here $\gamma_{i,j} = 1$ if, after removing the edge $e_i$, the edge $e_j$ and the vertex $s$ lie in one connected component, and $\gamma_{i,j} = 0$ otherwise. Next, $\delta_{i,j} = 1$ if $e_i$ and $e_j$ are cyclic edges, and $\delta_{i,j} = 0$ otherwise. Summation $\sum^{(1)}$ is taken over the non-end isthmuses $e_j$. The sum $\sum^{(2)}$ is taken over all unordered pairs of edges $\{e_i, e_j\}$, such that after removing these two edges, the graph $G = \Gamma \setminus \{e_i, e_j\}$ consists of $m$ isolated vertices and another connected component. The sum $\sum^{(3)}$ is taken over all unordered pairs of edges $\{e_i, e_j\}$, such that after removing isolated vertices from the graph $G = \Gamma \setminus \{e_i, e_j\}$, we obtain two connected components. The sum $\sum^{(4)}$ is taken over all unordered pairs of edges $\{e_i, e_j\}$, such that they are incident to a vertex of valence 2 (where again $G = \Gamma \setminus \{e_i, e_j\}$).

Here $\beta_1(G)$ is the first Betti number of the graph $G$.

The second term of the asymptotic expansion is connected with the graph structure. The graph can be recovered uniquely if the second term is known as a function of lengths in the case of a tree (see [3]).

This is joint work with A. A. Tolchennikov. The research was partially financially supported by the grant 16-11-10069 of the Russian Science Foundation.

**References**


Schrödinger–Poisson system involving a measure

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We discuss the following system of PDEs

\[\begin{align*}
(-\Delta)^s u + u + l(x)\phi u + w(x)|u|^{k-1}u &= \mu \text{ in } \Omega, \\
(-\Delta)^s \phi &= l(x)u^2 \text{ in } \Omega, \\
u &= \phi = 0 \text{ in } \mathbb{R}^N \setminus \Omega,
\end{align*}\]

where \((-\Delta)^s\) is the fractional Laplacian operator and \(\Omega\) is a bounded domain in \(\mathbb{R}^N\) with boundary \(\partial \Omega\). We guarantee the existence and uniqueness of a solution to (1) when the nonlinearity \(g(u) = u + l(x)\phi u + w(x)|u|^{k-1}u\) in the problem (1) satisfies a "subcritical integrability condition."

The main results of this paper is the following.

Theorem. The problem

\[\begin{align*}
(-\Delta)^s u + u + l(x)\phi u + w(x)|u|^{k-1}u &= \mu \text{ in } \Omega, \\
(-\Delta)^s \phi &= l(x)u^2 \text{ in } \Omega, \\
u &= \phi = 0 \text{ in } \mathbb{R}^N \setminus \Omega,
\end{align*}\]

admits a unique very weak solution \((u, \phi)\) corresponding to \(\mu \in m(\Omega, \rho^2)\). Further,

\[-G[\mu_-] \leq u \leq G[\mu_+] \text{ a.e. in } \Omega\]

where \(l, w\) are nonnegative bounded functions on \(\Omega\), \(1 < k, 0 < s < 1\), \(\Omega \subset \mathbb{R}^N\) is a bounded domain, \(\mu\) is a nonnegative bounded Radon measure, \(g(x,t) = t + l(x)\phi t + w(x)|t|^{k-1}t\) is a continuous, nondecreasing function satisfying \(rg(r) \geq \beta r^\beta\) for all \(r \in \mathbb{R}\), and for each \(x \in \Omega\), \(\int_1^\infty (g(x,s) - g(x,-s))s^{1-k}s + \beta \, ds < \infty\). Here \(\mu_-\), \(\mu_+\) are the positive and negative parts of the Jordan decomposition of \(\mu\). \(G[\cdot]\) being the Green’s operator corresponding to the fractional Laplacian \((-\Delta)^s\). The critical exponent is defined as

\[k(s, \beta) = \begin{cases} 
\frac{N}{N-2s} & \beta \in [0, \frac{N-2s}{N} \alpha), \\
\frac{N+\alpha}{N-2s+\beta} & \beta \in (\frac{N-2s}{N} \alpha, \alpha]
\end{cases}\]

for \(N \geq 2, 0 < s < 1, 0 < \beta < \alpha\).

References
Generalized exponential type solutions
to the Kolmogorov–Feller equation: forward
and backward in time motion, vanishing diffusion

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WKB solutions were popular in the theory of linear hyperbolic PDE. There was a lot of reasons for that, one of them is the possibility of parametrix construction.

At the same time, nonoscillating WKB-type solutions in the theory of linear parabolic equations are not so popular. The direct analog between these two theories has been done by Yu. Kifer (for small times) and by V. P. Maslov in the large.

The last theory is spiritually similar to Maslov’s canonical operator theory and uses integral transformation with the heat kernel instead of Fourier transform in hyperbolic theory.

But some special features of parabolic equations allow to avoid any integral transformations. The price of that is the using of nonsmooth solutions to Hamilton-Jacobi equations (viscosity solutions) and transport equations with singular coefficients. The last problem relates to delta-shock solutions in the theory of hyperbolic conservation laws, which was developed in part by V. Shelkovich and the author.

Using this approach, we can construct WKB-type solutions by means of characteristics only. More that, this approach can be extended to construction solutions of a Cauchy problem backward in time.

Another application is degenerated parabolic equations. As an example, we consider asymptotic of the fundamental solution to the equation

$$u_t - \varepsilon x^2 u_{xx} = 0,$$

where $\varepsilon \to +0$ is small parameter.

References


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Elliptic pseudo-differential boundary value problems and the inverse problem of magneto-electroencephalography

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Contrary the already prevailing for several decades opinion about the incorrectness of the inverse-MEEG problems (see, for example: Sheltraw, D. and Coutsias, E. (2003) Journal of Applied Physics, 94 (8), 5307-5315), the report will show that this problem is absolutely correct. Namely: under the condition of reconstruction electromagnetic field according to its measurement in the final set of points $x_k$ on the head of the patient the inverse MEEG problem has, and the only solution in a special class of functions (different from those considered by biophysicists). Moreover, the operator of this problem realizes an isomorphism of the corresponding function spaces. The solution has the form $q = q_0 + p_0 \delta |_{\partial Y}$, where $q_0$ is an ordinary function defined in the domain of the region $Y$ occupied by the brain, and $p_0 \delta |_{\partial Y}$ is a $\delta$-function on the boundary of the domain $Y$ with a certain density $p_0$. The functions $p_0$ and $q_0$ are interrelated and explicitly depend on the reconstructed electromagnetic field. Its reconstruction reduces to a finite-dimensional problem of minimizing a quadratic functional and revealing “essentially” various minimizing elements. The latter question echoes the analogous problem for the inverse problem of an equilibrium plasma in a tokamak [1].

This result [2] was obtained due to the fact that: 1) Maxwell’s equations are taken as a basis; 2) a transition was made to the equations for the potentials of the magnetic and electric fields; 3) the theory of boundary value problems for elliptic pseudodifferential operators with an entire index of factorization is used. This allowed us to find the correct functional class of solutions of the corresponding integral equation of the first kind: the solution has a singular boundary layer in the form of a delta function (with some density) at the boundary of the domain.

References
Finding algebraic invariants and algebraically invariant solutions

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Nonlinear partial and ordinary differential equations arise in a variety of processes and phenomena in physics, biology, chemistry, economics, etc. In this talk we shall discuss the concept of algebraic invariants and algebraically invariant solutions for autonomous ordinary differential equations and systems of autonomous ordinary differential equations. The motivation of introducing the notion of algebraic invariants lies in the fact that a variety of known exact solutions of autonomous ordinary differential equations and a great number of traveling wave solutions of famous partial differential equations are in fact algebraically invariant solutions.

Our aim is to present a novel method of finding algebraic invariants in explicit form. The basic idea of the method is to use Puiseux series satisfying a non–autonomous ordinary differential equation related to the original equation [2, 3]. A great advantage of our approach lies in the fact that the method under consideration is able to give not some but all algebraically invariant solutions.

As an example, we shall classify all algebraic invariants and algebraically invariant solutions for traveling wave reductions of the modified Burgers–Kolmogorov equation

\[ u_t = Du_{xx} + \beta u^2 u_x + \delta u(1 - u^2), \]

the Newell – Whitehead – Segel equation

\[ u_t = Du_{xx} + \beta u + \delta u^m, \quad m \in \mathbb{N}, \quad m \geq 2, \]

and some other physically relevant partial differential equations.

Further, using our approach we shall solve completely the problem of Liouvillian integrability for dynamical systems related to the traveling wave reductions of aforementioned partial differential equations [1, 2, 3].

References
Lagrangian manifolds related to Bessel functions, and their applications

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We discuss the question about the relationship between the Bessel functions and Lagrangian manifolds in the phase space. We present such 2-D regular and 1-D singular Lagrangian manifolds $\Lambda$ that the Maslov canonical operator $K^h_{\Lambda}A$ acting to appropriate functions $A$ on $\Lambda$ gives the functions expressed via the Bessel functions of the complex arguments. Also we discuss the new integral representations of the Maslov canonical operator near the Lagrangian singularities and the Fock quantization of canonical transforms in the problems connected with the case of singular Lagrangian manifolds. We discuss the application of constructed asymptotic formulas in the beam propagation theory and in problems for the wave equations with degenerated velocity appearing in the water wave theory.

Singular & singularly perturbed differential systems and their multiple resurgence

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The following may be taken as a model of a doubly singular differential system

$$0 = \epsilon t^2 \partial_t y^i + \lambda_i y^i + b^i(t, \epsilon, y^1, \ldots, y^\nu) \quad (1 \leq i \leq \nu) \quad (1)$$

since it is not only singular (in the time variable $t$) but also singularly perturbed (by the small parameter $\epsilon$). Under suitable assumptions on the coefficients $b^i(t, \epsilon, y)$, it is also doubly resurgent, meaning that the formal solutions of (1)), whether expanded in powers of $t := 1/z$ or $\epsilon := 1/x$, lead to convergent analytic germs in the corresponding Borel planes $\zeta$ or $\xi$, with all the trade-mark features of resurgence: endless analytic continuation; isolated singularities; remarkable self-reproduction properties.

$$t \sim 0 \Rightarrow z := \frac{1}{t} = \text{critical variable} \Rightarrow \zeta\text{-Borel plane } \left( \sum \alpha_n z^{-n} \mapsto \sum \alpha_n \frac{\zeta^{n-1}}{(n-1)!} \right)$$

$$\epsilon \sim 0 \Rightarrow x := \frac{1}{\epsilon} = \text{critical param.} \Rightarrow \xi\text{-Borel plane } \left( \sum \beta_n x^{-n} \mapsto \sum \beta_n \frac{x^{n-1}}{(n-1)!} \right)$$

But whereas the now classical resurgence in $z$ (equational resurgence) is well-understood and easily described, the loosely dual but incomparably more complex resurgence in $x$ (co-equational resurgence) was long thought to defy a unified treatment. As we shall attempt to show, such pessimism is unwarranted. Nothing stands in the way of a general approach: one can produce a complete system of resurgence equations that subsume all the properties of the $x$-expansions.
Two features stand out: the centrality of combinatorics to the theory, and the crucial part played by the so-called tessellation coefficients \( t_{s(t_1, \ldots, t_r)} \). The latter are piece-wise constant functions on \( \mathbb{C}^{2r} \), paradoxically defined as superpositions of hyperlogarithms. These coefficients possess no end of remarkable properties. Mainly, they encapsulate a sort of universal geometry that rules co-equational resurgence, and play there roughly the same role as the Stokes constants do in equational resurgence.

Large-time decay of solutions to the Zakharov–Kuznetsov equation

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The two-dimensional Zakharov–Kuznetsov equation

\[ u_t + u_x + u_{xxx} + u_{xyy} + uu_x = 0 \]  

(1)

is a model of nonlinear waves in dispersive media propagating in one preassigned \((x)\) direction with deformations in the transverse \((y)\) direction. In the case of the initial value problem, the well-known conservation law in the space \( L^2(\mathbb{R}^2) \) excludes the large-time decay of solutions in this space. From the physical point of view, initial-boundary value problems in domains where the variable \( y \) is considered on a bounded interval seem to be natural. Then certain internal dissipation can ensure such large-time decay.

Let \( I_1 = \mathbb{R} \), \( I_2 = \mathbb{R}_+ \), \( I_3 = (0, R) \), and \( \Sigma_j = I_j \times (0, L) \). Consider three initial-boundary value problems, posed on the domains \( \Sigma_j \) with the initial data \( u_0(x, y) \), homogeneous Dirichlet boundary conditions \( u|_{y=0} = u|_{y=L} = 0 \), homogeneous Dirichlet boundary condition on the left side \( u|_{x=0} = 0 \) for \( j = 2 \) and \( j = 3 \), and additional boundary conditions on the right side \( u|_{x=R} = u_x|_{x=R} = 0 \) for \( j = 3 \).

**Theorem 1.** For \( j = 1 \) and \( j = 2 \) there exists an \( L_0 > 0 \) such that for any \( L \in (0, L_0) \) there exist \( \alpha_0 > 0 \), \( \varepsilon_0 > 0 \), and \( \beta > 0 \) such that if \( (1 + e^{\alpha x}) u_0 \in L_2(\Sigma_j) \) for \( \alpha \in (0, \alpha_0] \) and \( \| u_0 \|_{L_2(\Sigma_j)} \leq \varepsilon_0 \), then there exists a weak solution of the corresponding problem for equation (1) satisfying the inequality

\[ \| e^{\alpha x} u(t, \cdot, \cdot) \|_{L_2(\Sigma_j)} \leq e^{-\alpha \beta t} \| e^{\alpha x} u_0 \|_{L_2(\Sigma_j)} \quad \forall t \geq 0. \]

**Theorem 2.** Let \( u_0 \in L_2(\Sigma_3) \), and let

\[ \pi^2 \left( \frac{3}{R^2} + \frac{1}{L^2} \right) > 1. \]

Then there exist \( \varepsilon_0 > 0 \) and \( \beta > 0 \) such that if \( \| u_0 \|_{L_2(\Sigma_3)} \leq \varepsilon_0 \), then there exists a weak solution of the corresponding problem (1) satisfying the inequality

\[ \| u(t, \cdot, \cdot) \|_{L_2(\Sigma_3)} \leq \sqrt{1 + R e^{-\beta t}} \| u_0 \|_{L_2(\Sigma_3)} \quad \forall t \geq 0. \]

In the cases of \( j = 2 \) and \( j = 3 \) these weak solutions are unique in certain classes.

This work was supported by RFBR grants 17-01-00849, 17-51-52022, 18-01-00590.
References

Quasiclassical asymptotics for solutions to difference equations with two close turning points

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In the complex plane we study analytic solutions Ψ to the difference equation
\[ Ψ(z + h) = M(z)Ψ(z), \quad z ∈ \mathbb{C}, \]  
where \( h > 0 \) is a parameter, and \( M \) is a given \( SL(2, \mathbb{C}) \)-valued analytic function.

We focus on the asymptotics of \( Ψ \) as \( h \to 0 \). Formally, \( Ψ(z + h) = \exp(h \frac{d}{dz})Ψ(z) \), and \( h \) appears in front of a derivative. So, it can be regarded as a quasiclassical parameter.

To study quasiclassical asymptotics of analytic solutions to ordinary differential equations on the complex plane, for example, to the Schrödinger equation
\[ -h^2 ψ''(z) + v(z)ψ(z) = Eψ(z) \]  
where \( v \) is an analytic potential, and \( E ∈ \mathbb{C} \) is the spectral parameter, one uses the well-known method often called the complex WKB method, see, e.g., [2, 7]. A version of the complex WKB method for difference equations was developed in [1, 4, 5, 6].

As \( h \to 0 \) solutions to (1) have a standard simple asymptotic behavior if \( \text{Tr} M(z) \neq ±2 \). We call the points where \( \text{Tr} M(z) ∈ \{-2, +2\} \) turning points as they play the same role as the turning points for the differential equation (2).

A.Fedotov and F.Klopp studied solutions to the difference Schrödinger equation
\[ ψ(z + h) + ψ(z - h) + v(z)ψ(z) = Eψ \]  
near simple turning points (defined by the equations \( E - v(z) = ±2 \)), see [3].

In the framework of the complex WKB method we study solutions to (1) in the case of two coalescent turning points. A similar analysis was carried out by V.Buslaev and A.Fedotov for the Harper equation, i.e., equation (3) with \( v(z) = \cos(z) \), but this never was published.

Our work was supported by RFBR under the grant No. 17-01-00668-a.
Topological billiards and integrable Hamiltonian systems

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When analyzing quite a few topological invariants (so-called marked molecules) calculated so far for various integrable billiards and other integrable systems with two degrees of freedom, the author stated a conjecture consisting of five parts A, B, C, D, and E. The talk will also describe the proof of two parts of a conjecture obtained jointly with V. V. Vedyushkina (Fokicheva) and I. S. Kharcheva.

Consider nondegenerate (Bott) integrable systems with two degrees of freedom, their three-dimensional isoenergetic surfaces, and the corresponding Liouville foliations.

**Conjecture A. (Atoms).** Any bifurcation of two-dimensional Liouville tori in an isoenergetic 3-manifold of any integrable nondegenerate system with two degrees of freedom can be modelled by integrable billiards. In other words, any orientable 3-atom can be realized as an appropriate billiard.

**Conjecture B. (Rough molecules).** Any so-called rough molecule can be modelled by an integrable billiard.

**Conjecture C. (Marked molecules).** Any marked molecules (i.e., Fomenko–Zieschang invariants) can be modelled by integrable billiards. In other words, all Liouville foliations of nondegenerate integrable systems on isoenergetic 3-surfaces are Liouville equivalent to the corresponding foliations of some billiard.

**Conjecture D.** Any closed three-dimensional isoenergetic surface of any integrable nondegenerate system with two degrees of freedom can be realized as an isoenergetic surface of some integrable billiard. This conjecture is a special case of Conjecture C, and so it is true provided that Conjecture C is. Recall that, according to a theorem due to A. V. Brailov and A. T. Fomenko, the class of isoenergetic 3-manifolds coincides with the class of graph-manifolds (Waldhausen manifolds).

References

Conjecture E. According to Conjecture C, there exists a large class of Liouville foliations that can be realized as integrable topological billiards. Then the Fomenko–Zieschang invariant for this class is “equivalent” to the corresponding billiard. More precisely, given an integrable billiard, consider it as a two-dimensional CW-complex in which the boundaries of 2-cells are formed by edges of the billiard and segments of focal lines. Then there exists a one-to-one correspondence between marked molecules (up to a fiberwise equivalence of Liouville foliations) and integrable billiards (up to cellular homeomorphisms of two-dimensional CW-complexes).

References

Asymptotic stability of an evolutionary nonlinear Boltzmann-type equation

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Some problems of the mathematical physics can be written as differential equations for functions with values in the space of measures. The vector space of signed measures doesn’t have good analytical properties. For example, this space with the Fortet–Mourier metric is not complete. There is the method to overcome this problem. We may restrict our equations to some complete convex subsets of the vector space of measures. This approach seems to be quite natural and it is related to the classical results concerning differential equations on convex subsets of Banach spaces (see [2]). The main purpose of our lecture is to show that the Kantorovich–Rubinstein maximum principle combined with the LaSalle invariance principle allow us to find new sufficient conditions for the asymptotic stability of solutions of the following version of the nonlinear Boltzmann-type equation.

\[
\frac{d\psi}{dt} + \psi = P \psi \quad \text{for} \quad t \geq 0
\]  \hspace{1cm} (1)

with the initial condition
\[
\psi(0) = \psi_0,
\]
where \(\psi_0 \in \mathcal{M}_1(\mathbb{R}_+)\) and \(\psi: \mathbb{R}_+ \to \mathcal{M}_{sig}(\mathbb{R}_+)\) is an unknown function. Moreover \(P\) is the collision operator acting on the space of probability measures. The collision operator \(P\) is a convex combination of \(N\) operators \(P_1, \ldots, P_N\), where \(P_k\) for \(k \geq 2\) describes the simultaneous collision of \(k\) particles and \(P_1\) the influence of external forces.

We will show that if our equation has a stationary measure \(\mu_*\) such that \(\text{supp} \mu_* = \mathbb{R}_+\), then this measure is asymptotically stable with respect to the Kantorovich–Wasserstein metric.
The open problem related with characterization of the stationary measure $\mu_*$ of the equation (1) will end the talk.

References


**Hochshild’s method for describing the Mackenzie obstruction to the construction of a transitive Lie algebroid**

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The Mackenzie obstruction is a three dimensional class of cohomologies ([1], 7.2.12, pp. 277) whose triviality is provided by the existence and construction of the transitive Lie algebroid on the manifold $M$ if we are given the set a data:

1. The local trivial bundle $L$ will typical fiber isomorphic to the finite-dimensional Lie algebra $g$ and structural group of all automorphisms $Aut(g)$ of algebra $g$, denoted by $LAB$.

2. Coupling between the tangent bundle $TM$ and bundle $LAB$ in the form of homomorphism

$$\Gamma^\infty(\Xi) : \Gamma^\infty(TM) \rightarrow \Gamma^\infty(\mathcal{D}_{out}(L))$$

of the cut space as finite-dimensional Lie algebras.

Following the paper [2], we formulate the problem of description of Mackenzie obstruction to construction of transitive Lie algebroid by the set of data: 1) $LAB$ bundle $L$ on a smooth manifold $M$ and 2) coupling $\Xi$ between the bundle $L$ and tangent bundle $TM$.

According to Hochschild [2] we consider the set of all bundles ($LAB$) equipped with the coupling $\Xi$ with tangent bundle $TM$, moreover with fixed modulus $ZL$ over the Lie algebra $\Gamma^\infty(TM)$ of vector fields. In the case of Lie algebras in Hochschild’s work ([2], pp. 698) it was shown that in such a set it is possible to set up the structure of linear space, and the mapping determined by the Mackenzie obstruction is a linear monomorphism.

Our task is to take the Hochschild construction to the case of transitive Lie algebroids and to prove the similar theorem: the set of all Mackenzie obstructions for
bundles \((LAB)\) \(L\) with coupling \(\Xi\) with one and the same center \(ZL\) as a modules over the algebra of vector fields \(\Gamma^\infty(TM)\) generates a linear subspace in the group \(H^3(M; ZL)\).

The second task is to generalize Hochschild’s theorem that states that this space coincides with the group \(H^3(M; ZL)\) \([2]\), pp. 716, Theorem 3.1, \([3]\), pp. 777, Theorems 4,5).

References


Derivation of the Benjamin-Ono equation at construction of the triple-deck structure in problems of a fluid flow along a plate with small irregularities on the surface

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We consider a viscous fluid flow problem along a semi-infinite plate with small periodic irregularities on the surface for large Reynolds numbers. Depending on the scale of irregularities, the asymptotic solution of the problem under study has a double-deck or a triple-deck boundary layer structure.

In both cases of the boundary layer structure, the flow in the near-wall region is described by a Prandtl-type system of boundary layer equations with induced pressure (with some differences in the boundary conditions). A numerical simulation shows that the behavior of the flow in the near-wall region is similar in the both cases (the difference lies in different values of the critical amplitude at which the laminar flow becomes a vortex flow).

It is of interest to study the equations describing the velocity oscillations in the boundary layers. In the double-deck case, it is a Rayleigh-type equation which is considered on the semi-infinite cylinder, and the lower boundary condition is the trace at infinity of the velocity in the near-wall region. In the triple-deck case, it is the Laplace equation on the semi-infinite cylinder, but the lower boundary condition is a solution of a Benjamin–Ono-type equation. However, we show that the flow satisfies the Benjamin–Ono-type equation automatically, and the “solution” of this equation can be determined from the trace at infinity of the velocity in the near-wall region.

References

On existence of time-global solutions for parabolic equations

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The symbol $E$ denotes the expectation and $P$, the probability. Consider in $\mathbb{R}^n$ the parabolic equation

$$\frac{\partial}{\partial t}u = Au \tag{1}$$

where $A$ is an autonomous strictly elliptic operator with $C^\infty$ coefficients without constant term. In coordinates the operator $A$ is represented in the form $a^i \frac{\partial}{\partial x^i}+\alpha^{ij} \frac{\partial^2}{\partial x^i \partial x^j}$ where $a^i$ are coordinates of some vector $a$ and the matrix $(\alpha^{ij})$ is a symmetric $n \times n$ matrix $\alpha$ of a certain $(2, 0)$-tensor field. Since $A$ is strictly elliptic, $\alpha$ is non-degenerate and we can find a unique $n \times n$ matrix $A$ such that $\alpha = AA^*$ where $A^*$ is the conjugate matrix to $A$. Thus we can construct a stochastic differential equation in Ito form

$$d\xi(t) = a(\xi(t))dt + A(\xi(t))dw(t) \tag{2}$$

for which $A$ plays the role of generator. Since the coefficients of (2) are smooth, it has unique at least local solutions for any initial time and value. Denote by $\xi(s)$ the stochastic flow generated by (2) and by $\xi_{t,x}(s)$ its orbit, i.e., the solution of (2) with initial condition $\xi_{t,x}(t) = x$. Consider a smooth function $u_0(x)$ on $\mathbb{R}^n$. It is a well-known fact (see, e.g. [1]), that if the flow exists up to $T > 0$, then the function $u(t,x) = Eu_0(\xi_{t,x}(T))$ is the solution of (1) on $[t, T]$ with condition $u(T,x) = u_0(x)$. Thus, if the flow $\xi(s)$ is complete, i.e., all the orbits exist up to $+\infty$, we can construct the above solutions for any $T > 0$.

We say that the flow $\xi(s)$ is continuous at infinity if for any $t < T$, any compact $K \subset \mathbb{R}^n$ and every sequence $x_i \to \infty$ the relation $\lim_{x_i \to \infty} P(\xi_{t,x_i}(T) \in K) = 0$ holds (see [2, 3]). Note that a sufficient condition for the flow to be continuous at infinity is the fact that it satisfies $C_0$-property.

Construct the direct product $\mathbb{R}^n_+ = [0, \infty) \times \mathbb{R}^n$ and introduce on $\mathbb{R}^n_+$ the flow $\xi_+(s) = (s, \xi(s))$. Obviously the generator of $\xi_+(s)$ has the form $\frac{\partial}{\partial s} + A$.

Recall that on the topological space $X$ the function $\varphi \colon X \to \mathbb{R}$ is called proper if the preimage of every relatively compact set in $\mathbb{R}$ is relatively compact in $X$.

**Theorem.** Let $\xi(s)$ on $\mathbb{R}^n$ be continuous at infinity. Then $\xi(s)$ is complete if and only if on $\mathbb{R}^n_+$ there exists a smooth positive proper function $\varphi$ such that $(\frac{\partial}{\partial s} + A)\varphi < C$ for a certain constant $C > 0$. 

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This work was supported by RFBR grant 18-01-00048.

References

Pairings for pseudodifferential symbols

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We give a uniform construction of the higher indices of elliptic operators associated to Alexander–Spanier cocycles of either parity in terms of a pairing between the $K$-theory and the cyclic cohomology of the algebra of complete symbols of pseudodifferential operators. While the formula for the lowest index of an elliptic operator $D$ on a closed manifold $M$ (which coincides with its Fredholm index) reproduces the Atiyah-Singer index theorem, our formula for the highest index of $D$ yields an extension to arbitrary manifolds of any dimension of the Helton–Howe formula for the trace of multicommutators of classical Toeplitz operators on odd-dimensional spheres. In fact, the totality of higher analytic indices for an elliptic operator $D$ amount to a representation of the Connes-Chern character of the $K$-homology cycle determined by $D$ in terms of expressions which extrapolate the Helton–Howe formula below the dimension of $M$.

Calculation of these higher indices can be obtained by comparison between various calculations of cyclic (co)homology of pseudodifferential symbols. We will also discuss how this comparison result leads construction of invariants of the algebraic $K$-theory of the algebra of pseudodifferential symbols.

References
Complex homotopy principle of Grauert and Gromov for algebras of pseudodifferential operators

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Let \( H := H(X; M) \) resp. \( C := C(X, M) \) be the set of holomorphic resp. continuous mappings from the Stein manifold \( X \) into the complex manifold \( M \). For each \( n = 0, 1, 2, \ldots \), the homotopy groups of \( H \) and \( C \) are isomorphic by deformations if \( M \) is finite dimensional, homogeneous or an elliptic bundle \([2, 1]\). We extend this result to the case where \( M \) is the set of Fredholm operators of kernel dimension smaller than \( k \) in some classes of pseudodifferential operators (e.g. of Hörmander) with spectral invariance. For finite dimensional matrices this is due to Gromov \([1]\). If the dimension of the kernel of the Fredholm operators is precisely \( k \) this is treated in \([4]\). Induced by \([3]\) we get connections to locally bounded algebras and their countable projective limits.

References


Eigenvalues for adiabatic problems in the presence of conical singularities

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The expression “adiabatic limit” refers to a family of Riemannian metrics \( g_h \) on a compact fibre bundle where lengths in the fibre direction are shrunk by a factor \( h \) compared to a fixed reference metric. An elementary example is the Euclidean metric on a \( 1 \times h \) rectangle. It is well-known that, under certain assumptions, the eigenvalues \( \lambda_k(h) \) of the Laplacian for \( g_h \) have a complete asymptotic expansion as \( h \to 0 \). We analyze the asymptotic behavior of the eigenvalues in certain non-smooth settings involving conical singularities, specifically to families of triangles degenerating to an interval.
Universal Euler characteristic of orbifolds
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The Euler characteristic is the only additive topological invariant for spaces of certain sort, in particular, for manifolds with some finiteness properties. A generalization of the notion of a manifold is the notion of a (real) orbifold called sometimes a $V$-manifold. We discuss a universal additive topological invariant of orbifolds: the universal Euler characteristic. It takes values in the ring $\mathcal{R}$ freely generated (as a $\mathbb{Z}$-module) by the isomorphism classes of finite groups. The ring $\mathcal{R}$ is the polynomial ring in the variables corresponding to the indecomposable finite groups. We also consider the universal Euler characteristic on the class of locally closed equivariant unions of cells in equivariant CW-complexes. We show that it is a universal additive invariant satisfying a certain “induction relation”. We give Macdonald type equations for the universal Euler characteristic for orbifolds and for cell complexes of the described type.

The talk is based on a joint work with I. Luengo and A. Melle-Hernández (Complutense University of Madrid). The work was supported by the RSF grant 16-11-10018.

Multi-normed spaces based on non-discrete measures, and their tensor products
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It was A. Lambert who discovered a new type of structures, situated, in a sense, between normed spaces and (abstract) operator spaces. His definition was based on the notion of amplification a normed space by means of spaces $\ell^n_2$. Afterwards several mathematicians investigated more general structure, “$p$-multi-normed space”, introduced with the help of spaces $\ell^n_p; 1 \leq p \leq \infty$. In the present talk we pass from $\ell_p$ to $L_p(X, \mu)$ with an arbitrary measure. This happened to be possible in the framework of the non-coordinate (“index-free”) approach to the notion of amplification, equivalent in the case of a discrete counting measure to the approach in mentioned articles.

Two categories arise. One consists of amplifications by means of an arbitrary normed space, and another one consists $p$-convex amplifications by means of $L_p(X, \mu)$. Each of them has its own tensor product of its objects whose existence is proved by a respective explicit construction. As a final result, we show that the “$p$-convex” tensor product has especially transparent form for the so-called minimal $L_p$-amplifications of $L_q$-spaces, where $q$ is the conjugate of $p$. Namely, tensoring $L_q(Y, \nu)$ and $L_q(Z, \lambda)$, we get $L_q(Y \times Z, \nu \times \lambda)$. 

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Global bifurcations on the two sphere

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Differential equations deal with the same matters as children do: pictures in the plane. If a picture related to a differential equation remains (topologically) the same after the equation is slightly perturbed, this equation is structurally stable. If it is not, abrupt changes of the corresponding picture may occur under a small perturbation. These abrupt changes are the subject of the bifurcation theory. This talk manifests the first steps of a new born branch of the bifurcation theory: global bifurcations on the two sphere. Bifurcations in generic one-parameter families were classified; the answer appeared to be quite unexpected. An important and non-trivial question “who bifurcates?” was answered. In all the previous works on the planar bifurcations, the result was described by a finite number of phase portraits that may occur under the perturbations of degenerate vector fields. In the global theory, this is no more the case. Even three-parameter families of vector fields on the two sphere may have numeric invariants, and six-parameter families may have functional invariants. These are joint results of the speaker and his collaborators: N. Goncharuk, D. Filimonov, Yu. Kudryashov, N. Solodovnikov, I. Schurov and others.

The development of the bifurcation theory will be outlined from the very beginning. Some open problems will be stated.

Complete semiclassical spectral asymptotics
for periodic and almost periodic perturbations
of constant operators

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Under certain assumptions we derive a complete semiclassical asymptotics

\[ e_{h,\varepsilon}(x, x, \lambda) \sim \sum_{n \geq 0} \chi_n(x, \tau, \varepsilon) h^{-d+n} \]

of the spectral function for a scalar operator

\[ A(x, hD, h) = A^0(hD) + \varepsilon B(x, hD, h), \]

where \( A^0 \) is an elliptic operator and \( B(x, hD, h) \) is a periodic or almost periodic perturbation.

In particular, a complete semiclassical asymptotics of the integrated density of states also holds. The proof (see [1]) combines the “Gauge transformation” of [2, 3] and the “hyperbolic operator method”.

Further, we consider generalizations.
Phase flow over 0-singularity

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The problem under discussion arose when the author offered the construction of one-dimensional double pendulum with special type of constraint [1]. The configuration space of this pendulum consists of two smooth lines in tangency. This situation means a geometrical uncertainty for trajectories of the motion equation. The series of experiments showed that there is no dynamical uncertainty [2]. Trajectories of motion always cross each other. However, absolutely unexpected was the fact that the lines has non zero curvature at the tangency point and the real trajectories on the state-space are not $C^2$-smooth. So the adequate mathematical model is needed to explain this phenomenon.

We offer the geometrical model of special embedding of vector bundle over singular manifold in $\mathbb{R}^3$. Consider the equation in $\mathbb{R}^3$

$$y^2 - \left( zx + \text{sign}(x)x^2e^{2\ln|x|}\right)^2 = 0.$$

1. For any section $x = \text{const}$ its components of connectedness are homeomorphic to $\mathbb{R}$. Hence, this manifold has natural structure of one-dimensional vector bundle.

2. All sections $z = \text{const} \neq 0$ are two transversal lines.

3. The section $z = 0$ is the base of bundle and is the lines in tangency of 1-order.

So, even a trajectory is smooth its projection on the base is not $C^2$-smooth. Now we can apply to this geometrical model different approaches to build the differential calculus over the base for modeling the above mechanical system.

References


Engineering of quantum Hamiltonians
by high-frequency laser fields

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Asymptotic methods are widely used in theory of dynamical systems with parameters quickly oscillating in time. In such cases, an effective perturbation theory in the inverse oscillation frequencies can be developed resulting in effective static Hamiltonians describing dynamics of slow degrees of freedom. This approach was developed first within classical mechanics [1, 2] and further modified to describe also quantum case [3, 4]. Physical realizations of the latter include ultracold gases in optical lattices [4] and solids under the effect of strong laser fields with photon energies much larger than the electron bandwidth [5, 6]. I will review applications of the perturbation theory to various quantum many-body systems including interacting electrons and phonons in a crystal [5] and spin systems at lattices [6]. The use of the strong laser field opens a way to manipulate very effectively the properties of physical systems, in particular, transforming antiferromagnetic exchange interaction to the ferromagnetic one, or creating exotic spin textures such as nanoskyrmion mosaics [6, 7, 8].

This work was supported by ERC Advanced Grant No. 338957 and by NWO via Spinoza Prize.

References
Simple localized solutions of the wave equation

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A review is given of simple explicit localized solutions of the wave equation

$$u_{xx} + u_{yy} + u_{zz} - \frac{1}{c^2} u_{tt} = 0, \quad c = const > 0.$$ 

Two classes of such solutions are under consideration.

First, we describe analytic solutions dependent on a certain parameter and becoming highly localized as it tends to infinity. These are the ones based on the complexified Bateman theory (see, e.g., [1, 2]) as well as those associated with the so-called “complex sources” [3, 4, 5, 6].

Second, we discuss simple solutions of the homogeneous wave equation, having a singularity at a running point. Much attention is paid to a detailed analytical investigation of the solution presented by Hörmander [7]. We found that it is a specification of the classical Bateman solution [8]. Also, we are concerned with a certain specialized complexified Bateman solution, having similar properties [9].

The author was supported by RFBR grant 17-01-00535.

References

On the rate of convergence as $t \to +\infty$ of the distributions of solutions to the stationary measure for the stochastic system of the Lorenz model for a baroclinic atmosphere

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We consider the system of equations for the quasi-solenoidal Lorenz model for a baroclinic atmosphere

$$
\frac{\partial}{\partial t} A_1 u + \nu A_2 u + A_3 u + B(u) = g, \quad t > 0,
$$
on the two-dimensional unit sphere $S$ centered at the origin of the spherical polar coordinates $(\lambda, \varphi)$, $\lambda \in [0, 2\pi)$, $\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\mu = \sin \varphi$. Here $\nu > 0$ is the kinematic viscosity, $u(t, x, \omega) = (u_1(t, x, \omega), u_2(t, x, \omega))^T$ is an unknown vector function and $g(t, x, \omega) = (g_1(t, x, \omega), g_2(t, x, \omega))^T$ is a given vector function, $x = (\lambda, \mu)$, $\omega \in \Omega$, $(\Omega, P, F)$ is a complete probability space,

$$
A_1 = \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta + \gamma I \end{pmatrix}, \quad A_2 = \begin{pmatrix} \Delta^2 & 0 \\ 0 & \Delta^2 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -k_0 \Delta & 2k_0 \Delta \\ k_0 \Delta & -(2k_0 + k_1 + \nu \gamma) \Delta + \rho I \end{pmatrix},
$$

$$
B(u) = (J(\Delta u_1 + 2\mu, u_1) + J(\Delta u_2, u_2), J(\Delta u_2 - \gamma u_2, u_1) + J(\Delta u_1 + 2\mu, u_2))^T.
$$

Also, $\gamma, \rho, k_0, k_1 \geq 0$ are numerical parameters, $I$ is the identity operator, $J(\psi, \theta) = \psi \theta_\mu - \psi_\mu \theta$ is the Jacobi operator and $\Delta \psi = ((1 - \mu^2)\psi_\mu + (1 - \mu^2)^{-1}\psi_{\lambda\lambda}$ is the Laplace-Beltrami operator on the sphere $S$. A random vector function $g = f + \eta$ is taken as the right-hand side of (1); here $f(x) = (f_1(x), f_2(x))^T$ and $\eta(t, x, \omega) = (\eta_1(t, x, \omega), \eta_2(t, x, \omega))^T$ is a white noise in $t$. For the existence of a unique stationary measure for the Markov semigroup defined by solutions of the Cauchy problem for (1) and for the exponential convergence of the distributions of solutions to the stationary measure as $t \to +\infty$, in [1] and the present work we obtain sufficient conditions on the right-hand side of (1) and the parameters $\nu, \gamma, \rho, k_0, k_1$:

$$
k_0 < \min_{i=1,2,\ldots,i_\ast} c(i), \quad c(i) = \frac{2}{(j(i) - \gamma)^2} \left( 3\nu j^2(i) (j(i) + \gamma) + \chi(j(i)) + \sqrt{(3\nu j^2(i) (j(i) + \gamma) + \chi(j(i)))^2 + (j(i) - \gamma)^2 (\nu^2 j^3(i) (j(i) + \gamma) + \nu j(i) \chi(j(i)))} \right),
$$

$$
\chi(y) = (k_1 + \nu \gamma)(y^2 + \gamma y) + \rho(\gamma + y), \quad j(y) = y(y + 1), \quad y \geq 0,
$$

$$
i_\ast = \left[ \frac{c_\ast}{2\nu} \left( \sqrt{1 + \frac{c_\ast}{\nu}} + 1 \right) \right]^{-1} \geq 1, \quad c_\ast = \begin{cases} 
0, & \text{if } \gamma \neq 2, \\
\frac{2}{\nu}, & \text{if } \gamma = 2,
\end{cases}
$$

A similar result is obtained for the equation of a barotropic atmosphere and the two-dimensional Navier–Stokes equations. A comparative analysis with some of the available related results is given for the latter.

The author was supported by the Russian Foundation for Basic Research (grant number 14-01-31110).
References

Fractional PDEs: brief introduction and new perspectives

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We shall discuss recent achievements in the analytic and probabilistic interpretation and extensions of the fractional PDEs of Caputo and Riemann–Liouville type and on the path integral representations for their solutions arising from this point of view. The main advantage of these path integral representations is their universality allowing to cover a variety of different problems in a concise unified way, and the possibility to yield solutions in a compact form that is explicitly stable with respect to the initial data and key parameters and is directly amenable to numeric schemes (Monte-Carlo simulation). Some of the author’s papers on the subject are cited.

References
Spectral geometry
of generalized smooth distributions

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We will discuss the Laplacians associated with a generalized smooth distribution on a smooth manifold $M$. By a generalized smooth distribution, we mean a locally finitely generated $C^\infty(M)$-submodule $\mathcal{D}$ of the $C^\infty(M)$-module $\mathcal{X}(M)$ of smooth compactly supported vector fields on $M$. Given a distribution $\mathcal{D}$, let $D_x$, $x \in M$, be a linear subspace of $T_xM$, which is the image of the evaluation map $ev_x: \mathcal{D} \to T_xM$, $X \mapsto X(x)$. A family $\{D_x : x \in M\}$ can be considered as a usual distribution on $M$, in general, of non-constant rank. If the dimension of $D_x$ is constant, $D$ is a smooth subbundle of the tangent bundle $TM$ and $\mathcal{D}$ is the $C^\infty(M)$-module of smooth sections of this bundle: $\mathcal{D} = C^\infty(M, \mathcal{D})$. In this case $\mathcal{D}$ is projective.

The fiber of $\mathcal{D}$ at $x \in M$ is the finite dimensional linear space $D_x = \mathcal{D}/I_x\mathcal{D}$, where $I_x = \{f \in C^\infty(M) : f(x) = 0\}$. We define a Riemannian metric on $\mathcal{D}$ as a family of inner products $\langle \cdot, \cdot \rangle_x$ on $D_x$, depending smoothly on $x \in M$ in some sense. We prove that such a Riemannian metric exists for an arbitrary distribution $\mathcal{D}$.

Given a smooth distribution $\mathcal{D}$ on a smooth manifold $M$, a Riemannian metric on $\mathcal{D}$ and a positive density $\mu$ on $M$, we construct the associated (horizontal) Laplacian $\Delta_\mathcal{D}$ as follows. First, we define the horizontal differential to be the operator $d_\mathcal{D} : C^\infty_c(M) \to C^\infty_c(M, \mathcal{D}^*)$ given by $d_\mathcal{D} = ev^* \circ d$, where $d : C^\infty_c(M) \to \Omega^1_c(M)$ is the de Rham differential and $ev^* : \Omega^1_c(M) \to C^\infty_c(M, \mathcal{D}^*)$ is induced by the evaluation maps $ev_x : \mathcal{D}_x \to T_xM$, $x \in M$. Then $\Delta_\mathcal{D}$ is the second order differential operator $\Delta_\mathcal{D} = d_\mathcal{D}^* \circ d_\mathcal{D} : C^\infty_c(M) \to C^\infty_c(M)$, where $d_\mathcal{D}^* : C^\infty_c(M, \mathcal{D}^*) \to C^\infty_c(M)$ is the adjoint of $d_\mathcal{D}$ with respect to natural inner products on $C^\infty_c(M)$ and $C^\infty_c(M, \mathcal{D}^*)$ defined by the Riemannian metric on $\mathcal{D}$ and the density $\mu$. If $M$ is compact, the horizontal Laplacian $\Delta_\mathcal{D}$ as an unbounded operator on the Hilbert space $L^2(M, \mu)$ with domain $C^\infty_c(M)$ is essentially self-adjoint.

A distribution $\mathcal{D}$ is called involutive if it is closed under Lie brackets. An involutive smooth distribution is called a singular foliation. In [1], I. Androulidakis and G. Skandalis constructed a longitudinal pseudodifferential calculus and the corresponding scale of longitudinal Sobolev spaces for an arbitrary singular foliation on a compact manifold.

Given a smooth distribution $\mathcal{D}$ on a compact manifold $M$, consider the smallest involutive $C^\infty_c(M)$-submodule $\mathcal{F}$ of $\mathcal{X}(M)$, which contains $\mathcal{D}$. It is generated by the elements of $\mathcal{D}$ and their iterated Lie brackets $[X_1, \ldots, [X_{k-1}, X_k]]$ such that $X_i \in \mathcal{D}$, $i = 1, \ldots, k$, for every $k \in \mathbb{N}$. Assume that $\mathcal{F}$ is a singular foliation (that is, it is finitely generated). We prove that the horizontal Laplacian $\Delta_\mathcal{D}$ is longitudinally hypoelliptic in the scale of longitudinal Sobolev spaces associated with $\mathcal{F}$.

If the distribution has constant rank, similar results were obtained in [2]. This is joint work with I. Androulidakis. It was supported by the Russian Foundation of Basic Research, grant no. 16-01-00312.

References
Entropy and renormalized solutions of elliptic equations with variable exponents of nonlinearities and measure data

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The paper focuses on the existence of entropy and renormalized solutions to Dirichlet problems for elliptic equations with variable exponents of nonlinearities of the form

$$\text{div } a(x, u, \nabla u) = |u|^{p_0(x)-2}u + b(x, u, \nabla u) + \mu, \quad x \in \Omega \subset \mathbb{R}^n,$$

where $\mu$ is a bounded Radon measure.

The problem in bounded domains was studied by T. Ahmedatt, M.B. Benboubker, E. Azroul, H. Chrarateh, M. El Moumni, H. Hjiaj, A. Touzani, C. Zhang, S. Zhou, and others. In the case of $p(x) = p$ and bounded domain $\Omega$ it was proved (see [1]) that $\mu$ is diffuse with $p$-capacity if and only if $\mu \in L^1(\Omega) + W^{-1,p'}(\Omega)$, i.e.

$$\mu = f - \text{div } f + f_0,$$

where $f \in L^1(\Omega)$, $f = (f_1, \ldots, f_n) \in (L^{p'}(\Omega))^n$, $f_0 \in L^{p'_0}(\Omega)$.

Here we consider Eq. (1) with $\mu = f - \text{div } f + f_0$, $f \in L^1(\Omega)$, $f = (f_1, \ldots, f_n) \in L^{p'_0}(\Omega)$, $f_0 \in L^{p'_0}(\Omega)$. It is assumed that the vector function $a(x, s_0, s)$ in Eq. (1) obeys the coercivity condition of the form

$$a(x, s_0, s) \cdot s \geq \sum_{i=1}^n |s_i|^{p_i(x)} - \phi(x), \quad \phi \in L^1(\Omega),$$

for all $s_0 \in \mathbb{R}$ and $s \in \mathbb{R}^n$ and for almost all $x \in \Omega$.

Set $\tilde{a}(x, s_0, s) = a(x, s_0, s) + f(x)$. Then Eq. (1) takes the form

$$\text{div } \tilde{a}(x, u, \nabla u) = |u|^{p_0(x)-2}u + b(x, u, \nabla u) + f(x) + f_0(x)$$

with the function $\tilde{a}(x, s_0, s)$ satisfying the coercivity condition of the form (2). Therefore, it suffices to consider Eq. (1) with $\mu = f + f_0 \in L^1(\Omega) + L^{p'_0}(\Omega)$.

We prove the existence of an entropy solution to the Dirichlet problem for Eq. (1) in an arbitrary domain $\Omega$. Previously, this was done only for bounded domains. In addition, we establish that the constructed solution is a renormalized solution of the problem.

This work was supported by RFBR grant 18-01-00428-a.

References

Some phenomena in the behavior of the eigenvalues of fractional differentiation operators

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This work is devoted to an exploration of some class of non-selfadjoint operators acting in a complex separable Hilbert space. We deal with the pair of complex separable Hilbert spaces $\mathcal{H}, \mathcal{H}^+$, with assumptions $\mathcal{H}^+ \hookrightarrow \mathcal{H}$. We consider a perturbation of the non-selfadjoint operator $T$ represented by an expression $W = T + A$ with certain assumptions relative to so-called main part - operator $T$ and a lower term - operator $A$, which is also a non-selfadjoint operator. Both of these operators act in $\mathcal{H}$. We suppose that there exists a linear manifold dense in $\mathcal{H}^+$ on which operators $T, A$ are well defined with their adjoint operators. Also we suppose that $T, A$ are strictly accretive. In opposite to the approach that was used in [1], where spectral properties of perturbation of selfadjoint and normal operators were studied, our considerations are founded on known spectral properties of the real component of non-selfadjoint operators. Having used the technic of sesquilinear forms theory we establish a compactness property of the resolvent, obtain asymptotic equivalence between the real component of the resolvent and the resolvent of the real component of non-selfadjoint operators. We conduct a classification of non-selfadjoint operators by belonging of their resolvent to Schatten-von Neumann’s class and formulate a sufficient condition for a completeness of the root vectors. Finally we obtain an asymptotic formula for the eigenvalues.

As an application of obtained theoretical results, we solve the eigenvalues problem for second order differential operators with fractional derivatives in the lower terms. This question is still relevant and many papers devoted to one, for instance the papers [2, 3, 4]. More precisely in [3] a spectrum problem for a second order differential operator with Riemman-Liuvile’s fractional derivative in the lower terms was considered, it was proved that the resolvent of this operator belongs to Hilbert-Shmidt class. We would like to research the multidimensional case which can be reduced to the cases considered in the works listed above. As a resume, it should be noted that results obtained in the theoretical part of our work allow us: to conduct a classification of second order differential operators with fractional derivatives in the lower terms by belonging of their resolvent to Schatten-von Neumann’s class, formulate the sufficient conditions for completeness of the root functions, to obtain the asymptotic formula for the eigenvalues.

References
Numerical solution of elliptic partial differential equations

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The present paper deals a fifth-order Newton-type iterative method for solving nonlinear equations. The derivative term has been removed from the proposed method using divided differences and it was proved that the method is fifth-order convergent in both the cases with and without derivative term. Various numerical comparisons are made in MATLAB to demonstrate the performance of the developed methods. Finally, fifth-order Newton-type iterative method has been applied to solve the nonlinear system of equations in finite element solution of nonlinear elliptic partial differential equations.

The use a priori analytic information to solve boundary-value problems of diffraction theory

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The talk is devoted to some application aspects of one problem on which Prof. B. Yu. Sternin was working: the theory of analytical continuation of solutions to differential equations of elliptic type. More precisely, we discuss methods for finding solutions to diffraction problems based on a priori information concerning analytical properties of solution. These methods include the method of auxiliary sources, the pattern equations method, the null field methods, the T-matrix method, and the extended boundary condition method. We also briefly discuss the problem of the recognition of mirror-like objects.

References
The simplest version of the Riemann-Hilbert correspondence is the statement, known for many decades, that the category of flat vector bundles on a smooth manifold \( M \) is equivalent to the category of representations of its fundamental group \( \pi_1(M) \). Recently a higher generalization of this statement was developed, [1], where the category of representations of \( \pi_1(M) \) was replaced by a differential graded category of infinity local systems on \( M \) and the category of flat vector bundles by a differential graded (dg) category of certain modules, called cohesive modules, over \( \Omega(M) \), the de Rham algebra of \( M \). The correspondence is given by a certain \( A_\infty \) functor.

The proof in loc.cit. is technically complicated and our original motivation was to understand it in simple terms, particularly keeping in mind that one side of the equivalence – the category of infinity local systems – is essentially the same as the more classical notion of a cohomologically locally constant (clc) dg sheaf, i.e. a dg sheaf whose cohomology forms an ordinary (graded) locally constant sheaf. Our approach is based on the observation that \( \Omega(M) \) is the global sections of the sheaf of de Rham algebras on \( M \) and the latter is a soft resolution of the constant sheaf \( \mathbb{R} \). Similarly, a dg module \( N \) over \( \Omega(M) \) could be sheafified and viewed as a module over the sheaf of de Rham algebras. Imposing suitable restrictions on \( M \), we show that the resulting sheaf of modules is quasi-isomorphic to a clc sheaf and that this procedure establishes an equivalence between the derived category of clc dg sheaves on \( M \) and a suitable homotopy subcategory of dg \( \Omega(M) \)-modules (such as cohesive \( \Omega(M) \)-modules). Our approach also generalizes to spaces more general than manifolds, e.g. simplicial complexes. Finally, we achieve a similar result working with the singular cochain complex of a topological space or a simplicial set, with values in rings other than \( \mathbb{R} \), e.g. \( \mathbb{Z} \).
The homotopy classification of transitive Lie algebroids

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Transitive Lie algebroids have specific properties that allow to look at the transitive Lie algebroid as an element of the object of a homotopy functor. Roughly speaking each transitive Lie algebroids can be described as a vector bundle over the tangent bundle of the manifold which is endowed with additional structures. Therefore transitive Lie algebroids admits a construction of inverse image generated by a smooth mapping of smooth manifolds. Then the non-abelian extension of transitive Lie algebroids due to K. Mackenzie (2005) can be used to construct a classifying space. The intention of my talk is to show the existence and classification of coupling between Lie algebra bundles and tangent bundles which plays a crucial role on classification of transitive lie algebroids.

References

On classification of the second order differential operators and differential equations

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We discuss a local classification of second order linear differential operators and corresponding the differential equations. Possibly Riemann [1] was apparently the first who analyzed this problem and found curvature as an obstruction to transform differential operators of the second order to operators with constant coefficients. In dimension two Laplace [2] found “Laplace invariants” which are relative invariants of subgroup of rescaling transformations of unknown functions, and Ovsyannikov [3] found the corresponding invariants. All invariants for hyperbolic equations in dimension two with respect to pseudogroup transformations, including also diffeomorphisms
of the base manifold, were found by Ibragimov [4]. For the case of ordinary differential operators, this was done by Kamran and Olver [5], and for the case of linear ordinary differential equations of any order relative invariants were found by Wilczynski [6]. We are going to consider the problem in all dimensions. The talk is based on joint work with Valeriy Yumaguzhin [7].

References

To spectral theory of Schrödinger and Dirac operators with point interactions

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Schrödinger and Dirac operators with δ-interactions are studied without any assumptions on the lengths of intervals. A connection between spectral properties of these operators and certain Jacobi matrices will be discussed. In particular, we show that Schrödinger operators and respective Jacobi operators are lower semibounded, non-negative, discrete, etc. only simultaneously. We will also discuss types (continuous, absolutely continuous, singular) of spectrum of these operators. A non-relativistic limit will be discussed too. Similar results have recently been obtained for Schrödinger operators on quantum graphs.

The talk is based on results of the papers [1, 2, 3] and recent publication [4]. The later publication was prepared with the support of the “RUDN University Program 5-100”.

References


**Publishing Mathematics with Birkhäuser**

Dorothy Mazlum  
Editor Mathematics Birkhäuser, Basel, Switzerland

In this presentation, we will give some background information on Birkhauser, highlight its mathematics program, and explain the publishing process from submission to publication. Moreover, current developments in the publishing industry, such as Open Access, will be discussed.

**Sobolev inequalities in arbitrary domains**

Vlaimir Maz’ya  
Linköping University

A theory of Sobolev inequalities in arbitrary open sets in $\mathbb{R}^n$ is presented. Boundary regularity of domains is replaced with information on boundary traces of trial functions and of their derivatives up to some explicit minimal order. The relevant Sobolev inequalities involve constants independent of the geometry of the domain, and exhibit the same critical exponents as in the classical inequalities on regular domains. Our approach relies upon new representation formulas for Sobolev functions, and on ensuing pointwise estimates which hold in any open set. This is a joint work with A.Cianchi.

**Boundary values and problems for elliptic $w$-operators on compact manifolds with fibered boundary**

Gerardo A. Mendoza  
Temple University, Philadelphia

I will present an overview of joint work with T. Krainer concerning boundary values of elliptic wedge operators on compact manifolds with fibered boundary, or complexes of cone operators (cone operators in the second case). I will first describe some aspects concerning the former. For these we have a complete theory in the case of first order operators which is in complete analogy with that of regular boundary value problems for elliptic operators. Concerning complexes, I will state a theorem
and illustrate some of the issues by contrasting the with the analogous problems in the case of an elliptic cone operator. If time permits, I will also give some applications.

**Klein–Gordon equation and growth of Lie algebras of vector fields**

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The notion of characteristic Lie algebra of a hyperbolic system of PDE was introduced by Leznov, Shabat and Smirnov in 1982 and in the recent years characteristic Lie algebras of various hyperbolic systems were actively studied by Zhiber’s and Habibullin’s schools. We will study the growth of the characteristic Lie algebra for the Klein–Gordon equation $u_{xy} = f(u)$.

We show that the characteristic Lie algebras of the Sinh–Gordon and Tzitzeica equations are isomorphic to the positive parts of two Kac–Moody affine algebras $A_1^{(1)}$ and $A_2^{(2)}$, respectively [2, 3]. Both these Lie algebras are in the classification list [1] of narrow naturally graded Lie algebras $\mathfrak{g} = \bigoplus_{i=1}^{+\infty} \mathfrak{g}_i$ of the “width 3/2" ($\dim \mathfrak{g}_i + \dim \mathfrak{g}_{i+1} \leq 3$) [1]. A Lie algebra $\mathfrak{g} = \bigoplus_{i=1}^{+\infty} \mathfrak{g}_i$ is called naturally graded if $[\mathfrak{g}_i, \mathfrak{g}_j] = \mathfrak{g}_{i+j}, i \geq j$. Shalev and Zelmanov proposed to call “narrow” those positively graded Lie algebras that have homogeneous subspaces $\mathfrak{g}_i$ at most two-dimensional. Narrow Lie algebras have slow linear growth (slow growth of the dimension of the subspace spanned by all commutators of length $n$ in generators of $\mathfrak{g}$).

The research was made under the support of the RSF grant No 14-11-00414.

References

**Semi-classical asymptotics for the two-dimensional radially symmetric Dirac equation**

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The current study is related to the question of how much the tip of a scanning tunneling microscope affects the measured values. Microscope measures the tunnel current between the tip and the crystal surface, and this current depends on the local density of states in the crystal. To perform the study the stationary Schrodinger and Dirac equations on the plane with a radially symmetric potential $U(r)$ are considered.
Using semiclassical asymptotic forms for the generalized eigenfunctions of these equations, the local density of states corresponding to these asymptotics is determined and analyzed. It is shown that in the case of the Schrodinger equation the tip significantly distorts the measured density only for small energies while in the case of the Dirac equation finite distortions appears for all energies.

**Derivations of group algebras and Hochschild cohomology**

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A description of the algebra of outer derivations of a group algebra of a finitely presented discrete group is given in terms of the Cayley complex of the groupoid of the adjoint action of the group. This task is a smooth version of Johnson's problem concerning the derivations of a group algebra. It is shown that the algebra of outer derivations is isomorphic to the group of the one-dimensional cohomology with compact supports of the Cayley complex over the field of complex numbers.

On the other hand the group of outer derivation is isomorphic to the one dimensional Hochschild cohomology of the group algebra. Thus the whole Hochschild cohomology group can be described in terms of the cohomology of the classifying space of the groupoid of the adjoint action of the group under the suitable assumption of the finiteness of the supports of cohomology groups.

The report presents the results partly obtained jointly with A. Arutyunov, and also with the help of A.I. Shtern.

**References**


**Completeness of perturbed systems of trigonometric functions in the space of Lebesgue integrable functions**

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The paper addresses the completeness of perturbed systems of sines and cosines with non integer indices and, moreover, complex, in the space of Lebesgue integrable functions. The criteria for the variation from the integer value have been found such that sine and cosine systems are complete.
This work was supported by a grant from the Russian Science Foundation (Project No. 16-11-10194).

References

Reduction of computation of the signature of a $G$-manifold in the simplest case

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In [3] it was defined a $C^*$-algebra signature for an algebraic Poincaré complex. This definition can be applied to the $l^2$-complex of an invariantly triangulated smooth co-compact $G$-manifold, for a discrete group $G$. In [2] it is shown that this is an (algebraic) homotopy invariant. In [1] it was shown that this is an algebraic bordism invariant. It will be discussed the task of reduction of the computation of this signature using the Conner–Floyd fixed points construction in the simplest case of a group $G$ with a single normal cyclic subgroup of prime order.

References
On absence of global positive solutions of correlated-noise elliptic inequalities
(noncoercive case)

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For the inequality
\[ \Delta u - \sum_{j=1}^{n} \alpha_j(x,u) \left( \frac{\partial u}{\partial x_j} \right)^2 \geq \omega(x,u) \] (1)
considered in \( \mathbb{R}^n \), the following result is true:

**Theorem 1.** Suppose that \( \alpha_j(x,s) \geq \frac{1}{s} \), \( j=1,\ldots,n \). Suppose that there exist a non-negative function \( k(x) \), a positive constant \( R_0 \), and a positive function \( \theta \) defined on \([R_0, +\infty)\) such that \( \lim_{t \to \infty} \theta(t) = \infty \), \( \frac{\theta(t)}{t^2} \) is a nonincreasing function, and the inequality \( k(x) \geq \frac{\theta(|x|)}{|x|^2} \) holds outside the ball \(|x| < R_0\). Suppose that there exists a constant \( p \) from \((1, +\infty)\) such that \( \omega(x,s) \geq k(x)s^p \). Then the inequality (1) has no global positive solutions.

This work was financially supported by the Ministry of Education and Science of the Russian Federation on the program to improve the competitiveness of Peoples’ Friendship University (RUDN University) among the world’s leading research and education centers in the 2016–2020, by the Russian Foundation for Basic Research (grant No. 17-01-00401), and by the President Grant for the Government Support of the Leading Scientific Schools of the Russian Federation, No. 4479.2014.1.

On fractional Neumann Laplacians in the half-space

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We consider different fractional Neumann Laplacians of order \( s \in (0, 1) \) on domains \( \Omega \subset \mathbb{R}^n \), namely, the **Restricted Neumann Laplacian** \( (-\Delta^N_{\Omega R})_s \), the **Semirestricted Neumann Laplacian** \( (-\Delta^N_{\Omega S_R})_s \) and the **Spectral Neumann Laplacian** \( (-\Delta^N_{\Omega S_p})_s \).

In particular, we are interested in the attainability of Sobolev constants for these operators when \( \Omega \) is a half-space.

The talk is based on the joint work with Roberta Musina [1]. This work was partially supported by RFBR, grant 17-01-00678.

**References**

Propagation and blowing-up of non-plane fronts in Burgers-type equations with modular advection

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We present recent results for initial boundary value problem for some classes of Burgers-type equations, for which we investigate moving fronts by using the developed comparison technique. These results are based on a further development of the asymptotic comparison principle (see [1] and references therein). For the considered initial boundary value problems the existence of moving fronts and their asymptotic approximation were investigated. The results were illustrated by the problem

\[ \frac{\partial^2 u}{\partial x^2} - A(u, x, t) \frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = f(u, x, t, \varepsilon), \quad x \in (0,1), \ 0 < t \leq T, \]

\[ u(0, t, \varepsilon) = u^0(t), \quad u(1, t, \varepsilon) = u^1(t), \quad t \in [0, T], \]

\[ u(x, 0, \varepsilon) = u_{init}(x, \varepsilon), \quad x \in [0,1]. \]

An asymptotic approximation of solutions with a moving front for specific forms of equation (1) in the case of modular advection term and nonlinear amplification is constructed. The considered problem extends results of the paper [2] for non-plane fronts case. The influence exerted by nonlinear amplification on front propagation and blowing-up is determined. The front localization and the blowing-up time are estimated.

This work was supported by the Russian Science Foundation (project No. 18–11–00042).

References

Lie algebroids and Lie algebra bundles

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A Lie algebroid is a vector bundle with a Lie structure on all vector sections associated with a bundle map called anchor to have the Leibniz identity. A transitive Lie algebroid is one in which the anchor is fiberwise surjective. Every Lie algebra bundle is a trivial Lie algebroid with a null anchor, so that the existence of a transitive Lie algebroid is important. For a Lie algebra, the problem of existence of a transitive Lie algebroid is equivalent to the Mackenzie obstruction to be trivial for the Mishchenko
Lie algebra bundle, i.e., the Lie algebra bundle with structural group consisting of all automorphisms for the Lie algebra with the topology that is discrete on its quotient group by the ad-automorphism subgroup. The Mackenzie obstruction is trivial for the case in which the Lie algebra is the direct sum of the quotient algebra and the center.

Whitney-Sullivan constructions for transitive Lie algebroids: the polynomial case

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H. Whitney [3] and D. Sullivan [2] considered several variants of cochain complexes defined over cell-like spaces by taking different notions of differential forms. They showed that the cohomology of those cochains complexes is isomorphic to the classical cohomology of the polytope underlying the cell-like space. Based in Rham–Sullivan theorem for cell spaces, it is proved that, for each transitive Lie algebroid $A$ on a triangulated compact manifold $M$, the inclusion mapping from the cochain algebra of all polynomial forms on $A$ to the cochain algebra of all piecewise smooth forms on $A$ induces an isomorphism in cohomology.

This work was supported by MICINN, Grant MTM2014-56950-P.

References

Dense quasi-free subalgebras of the Toeplitz algebra

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The Toeplitz algebra (i.e., the universal $C^*$-algebra $T$ generated by an isometry) has several interesting dense locally convex subalgebras, for example, the algebraic Toeplitz algebra $T_{alg}$ and the smooth Toeplitz algebra $T_{smth}$. Such subalgebras, as well as the Toeplitz algebra itself, play an important role in bivariant $K$-theory and in cyclic homology. The motivation for our talk comes from the observation (probably due to Ralf Meyer) that $T_{alg}$ is quasi-free in the sense of Cuntz and Quillen. On the other hand, $T$ is not quasi-free by a general result of Oleg Aristov. Now a natural question is whether or not $T_{smth}$ is quasi-free. To answer this question, we introduce a family $\{T_{P,Q}\}$ of dense locally convex subalgebras of $T$ indexed by Köthe power sets.
$P, Q$ satisfying some natural conditions. Our main result gives a sufficient condition (in terms of $P$ and $Q$) for $T_{P, Q}$ to be quasi-free. As a corollary, we show that the smooth Toeplitz algebra $T_{\text{smth}}$ and the holomorphic Toeplitz algebra $T_{\text{hol}}$ are quasi-free.

This is a part of a joint project with O. Aristov.

On the sum of narrow and compact operators

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Linear narrow operators in function spaces and vector lattices were studied by many authors (see [5]). The notion of the narrow operator was extended to the setting of lattice-normed spaces in [6]. It was proved in [3] that the sum of two narrow operators in generally is not a narrow operator. Nevertheless, some particular cases were investigated in [2, 3]. The aim of this short notes is to present a new result in this direction. For the standard information we refer to the monograph [1].

An element $y$ of a vector lattice $E$ is said to be a fragment of an element $x \in E$, if $y \perp (x - y)$. The notation $y \sqsubseteq x$ means that $y$ is a fragment of $x$. The set of all fragments of the element $x \in E$ is denoted by $F_x$. Two fragments $y, z$ of $x$ are said to be mutually complemented if $y \perp z$ and $x = y + z$.

Let $E$ be a vector lattice, and let $F$ be a real linear space. An operator $T : E \to F$ is said to be orthogonally additive if $T(x + y) = Tx + Ty$ for any disjoint elements $x, y \in E$.

Let $E$ be an atomless vector lattice and $F$ be a normed space. An orthogonally additive operator $T : E \to F$ is said to be narrow if for any $\varepsilon > 0$ and $x \in E$ there exist mutually complemented fragments $x_1, x_2$ of $x$ such that $\|T(x_1) - T(x_2)\| < \varepsilon$.

Let $E$ be a vector lattice. A net $(v_\alpha)_{\alpha \in \Lambda} \subset E$ is said to be laterally convergent to $v \in E$ if $v = (o) - \lim_{\alpha} v_\alpha$ and $|v_\beta - v_\gamma| \perp \|v_\gamma\|$ for all $\beta, \gamma \in \Lambda, \beta \geq \gamma$. Let $E$ be a vector lattice and $F$ be a normed space. An operator $T : E \to F$ is said to be:

- laterally-to-norm continuous provided $T$ sends laterally convergent nets in $E$ to norm convergent nets in $F$;
- $C$-compact if a set $T(F_x)$ is relatively compact in $F$ for every $x \in E$.

The following theorem is the main result of this notes.

**Theorem 1.** Let $E$ be an atomless vector lattice, $F$ be a Banach space, $S : E \to F$ be an orthogonally additive narrow operator and $T : E \to F$ be a laterally-to-norm continuous $C$-compact orthogonally additive operator. Then $R = S + T$ is a narrow operator as well.

This work was supported by RFBR (the grant number 17-51-12064).

**References**


Cyclic homology of crossed-product algebras

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There is a great amount of work on the cyclic homology of crossed-product algebras, by Baum–Connes, Brylinski–Nistor, Connes, Crainic, Elliott–Natsume–Nest, Feigin–Tsygan, Getzler–Jones, Nest, Nistor, among others. However, at the exception of the characteristic map of Connes from the early 80s, we don’t have explicit chain maps that produce isomorphisms at the level of homology and provide us with geometric constructions of cyclic cycles in the case of group actions on manifolds or varieties.

The aim of this talk is to present the construction of explicit quasi-isomorphisms for crossed products associated with actions of discrete groups. Along the way we recover and clarify various earlier results (in the sense that we obtain explicit chain maps that yield quasi-isomorphisms). In particular, we recover the spectral sequences of Feigin–Tsygan and Getzler–Jones, and derive an additional spectral sequence.

In the case of group actions on manifolds we have an explicit description of cyclic homology and periodic cyclic homology. In the finite order case, the results are expressed in terms of what we call “mixed equivariant homology”, which interpolates group homology and de Rham cohomology. This is actually the natural receptacle for a cap product of group homology with equivariant cohomology. As a result taking cap products of group cycles with equivariant characteristic classes naturally gives to a geometric construction of cyclic cycles. For the periodic cyclic homology we recover earlier results of Connes and Brylinski–Nistor via a Poincaré duality argument. For the non-periodic cyclic homology the results seem to be new. In the infinite order case, we fix and simplify the misidentification of cyclic homology by Crainic.

In the case of group actions on smooth varieties we obtain the exact analogues of the results for group actions on manifolds. In particular, in the special case of finite group actions on smooth varieties we recover recent results of Brodzki–Dave–Nistor via the construction of an explicit quasi-isomorphism.

This work was supported by Basic Research Grant 2016R1D1A1B01015971 of National Research Foundation of Korea.

References
Combinatorial Ricci flow for degenerate metrics

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In 1998 R. Hamilton published a paper in which he proved that the Ricci flow for any closed orientable surface not diffeomorphic to the sphere for any metric converges to a metric of constant curvature. He proved the same statement for a sphere provided an initial metric has positive Gauss curvature. Few years later B. Chow proved the convergence of the Ricci flow to a metric of constant curvature on a sphere for any initial metric.

In 2003 B. Chow and F. Luo introduced the combinatorial Ricci flow for triangulated surfaces. They gave a complete description of the asymptotic behaviour of the solution of the combinatorial Ricci flow under certain assumption. Both the Euclidean and hyperbolic background geometry were considered. Their theory uses Thurston's circle packing metrics on triangulated surfaces with weights on the edges of the triangulation. Under assumption that weights belong to $[0,1]$ the Chow-Luo's conditions of convergence of the Ricci flow coincide with the Thurston's condition of existence of a constant curvature metric.

In our papers with R. Pepa we have weaken condition of non-negativity of the weights in such a way that the theorem about convergence of the combinatorial Ricci flow for any initial condition still holds. On the other hand, we found several examples of triangulated surfaces with some negative weights such that under the Ricci flow some initial metrics degenerate. At the same time numerical simulation shows some regularity in behaviour as time tends to infinity.

In the present talk we give a definition of degenerate circle packing metrics and prove under conditions similar to the Chow-Luo's conditions the convergence of the combinatorial Ricci flow to a constant curvature metric for any initial condition.
Semiclassical theory
of electronic Veselago lensing in graphene

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The motion of low-energy charge carriers in graphene is governed by the Dirac equation. Electron-hole interfaces in this material are able to focus electrons and therefore behave as electronic analogs of Veselago lenses [1]. The intensity near the focus can be expressed in terms of the Green’s function. For quasi-one-dimensional interfaces, this Green’s function can be determined analytically. It is given by an integral expression, which we study using the stationary phase approximation. We observe that, even for large values of the dimensionless semiclassical parameter, the leading-order approximation in terms of the Pearcey function correctly predicts the position of the intensity maximum. We use this observation to study two sources of aberrations in graphene Veselago lenses. The first of these is the second-order term in graphene’s Hamiltonian, the so-called trigonal warping term. This term significantly modifies the classical trajectories [2] and destroys the ideal focus that is present for the Dirac equation [3]. Using the stationary phase approximation, we can however still obtain a quasi-analytical formula for the position of the intensity maximum, which is in very good agreement with numerical results. The second source of aberrations is the initial sublattice polarization. This polarization leads to a sideward shift of the focus [4], which we study by considering corrections to the leading-order approximation.

References
Elliptic functional differential equation with contracted and shifted argument

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Let $p > 1$, $K \subset \mathbb{R}^n$ be a compact set, and $\Omega \subset \mathbb{R}^n$ a bounded domain such that $p^{-1}\Omega - K \subset \Omega$. Consider also any regular complex-valued Borel measure $\nu$ concentrated on $K$, $\nu \in (C(K))^*$. By $\tilde{\nu}$ denote its characteristic function, $\tilde{\nu}(\xi) = \int_K e^{-ih\xi} d\nu(h)$.

Define an operator with contracted and shifted argument as follows:

$$Tu(x) = \int_K u(p^{-1}x - h) d\nu(h).$$

Lemma 1. The operator $T$ is a bounded linear operator in the Sobolev spaces $H^s(\Omega)$ $s \in \mathbb{R}$ with

$$\rho(T) = \rho(T; L_2(\mathbb{R}^n)) = p^{n/2} \lim_{m \to \infty} \sup_{\xi \in \mathbb{R}^n} |\tilde{\nu}(\xi)\tilde{\nu}(p\xi) \cdots \tilde{\nu}(p^{m-1}\xi)|^{1/m}.$$

If $\alpha \in \mathbb{C}$ with $|\alpha| < 1/\rho(T)$, then the operator $I + \alpha T$: $H^s(\Omega) \to H^s(\Omega)$ has a bounded inverse for all $s \geq 0$. If, in addition, $\alpha$ satisfies the stronger inequality $|\alpha| < 1/(p^s\rho(T))$ for some positive $s$, then the operator $I + \alpha T$: $H^{-s}(\Omega) \to H^{-s}(\Omega)$ is boundedly invertible as well.

We consider the boundary value problem

$$-\Delta(u(x) + \alpha Tu(x)) = f(x) \quad (x \in \Omega),$$
$$u|_{\partial\Omega} = 0,$$

where $f \in L_2(\Omega)$ and the (generalized) solution is understood as a function $u \in \hat{H}^1(\Omega)$ satisfying the identity

$$\sum_{j=1}^n ((u + \alpha Tu)_x, v_x)_L_2(\Omega) = (f, v)_{L_2(\Omega)} \quad (v \in \hat{H}^1(\Omega)).$$

Theorem 1. If $|\alpha| < p/\rho(T)$, then problem (1) and (2) has a unique solution for any function $f \in L_2(\Omega)$. If, in addition, $f \in H^k(\Omega)$ and $\partial\Omega \in C^{k+2}$ ($k$ nonnegative integer), then $u \in H^{k+2}(\Omega)$.

Some specific examples are considered. It is also shown that for large $\alpha$ problem (1) and (2) may have infinitely many generalized solutions for any function $f \in L_2(\Omega)$ (in the sense that these solutions form an infinite-dimensional linear manifold).

In [1], the theory of boundary value problems for elliptic differential-difference equations is constructed. Elliptic functional differential equations with contracted and expanded argument are studied in [2]. The principal limitation of the latter work is the assumption that all the contractions have the same center, which essentially narrows the range of problems considered. In the present talk, we study the equation with contractions whose centers are distributed over the domain.
Let $M = \mathbb{R}^n$ or possibly a Riemannian, non compact manifold. We consider semi-excited resonances for a $h$-pseudo-differential operator $H(x, hD_x; h)$ on $L^2(M)$ induced by a nondegenerate periodic orbit $\gamma_E$ of semi-hyperbolic type, contained in some energy surface $\Sigma_E$. We may think of $H(x, hD_x; h)$ as Schrödinger operator with Stark effect, with $M = \mathbb{R}^n$, or $H(x, hD_x; h)$ as the geodesic flow on an axially symmetric manifold $M$, extending Poincaré example of Lagrangian systems with 2 degree of freedom. By semi-hyperbolic, we mean that the linearized Poincaré map $P_E$ associated with $\gamma_E$ has at least one eigenvalue of modulus greater (or less) than 1, and by non-degenerate that 1 is not an eigenvalue. We look for semi-excited resonances near $E$, i.e. with imaginary part of magnitude $h^s$, with $0 < s < 1$. It is known that these resonances are given by the zeroes of a determinant $\zeta(z, h) = \det(I - M(z, h))$ where $M(z, h)$ is the monodromy operator. We make here this result more precise, in providing a first order asymptotics for Bohr-Sommerfeld quantization rule associated with $\gamma_E$ in terms of the (real) longitudinal and (complex) transverse quantum numbers, including the action integral, the sub-principal 1-form and Gelfand-Lidskii (or Cohnley–Zehnder) index. This index enters only when $P_E$ has some eigenvalue of modulus 1. In the latter case, a theorem of Lewis and Birkhoff shows that an infinite number of periodic orbits cluster near $\gamma_E$, with periods approximately multiples of the primitive period of $\gamma_E$. Extending some result by Sjöstrand and Zworski, we propose also a generalized Poisson formula expressing $\text{Tr} f(H/h)$ microlocally near $\gamma_E$, as a truncated series (depending on $h$) of integrals of the form $\text{Tr} \int f(z/h)M(z, h)^K \frac{dK}{K!}M(z, h)dz$. 

References

Quasi-classical quantum maps
of semi-hyperbolic type
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Let $M = \mathbb{R}^n$ or possibly a Riemannian, non compact manifold. We consider semi-excited resonances for a $h$-pseudo-differential operator $H(x, hD_x; h)$ on $L^2(M)$ induced by a nondegenerate periodic orbit $\gamma_E$ of semi-hyperbolic type, contained in some energy surface $\Sigma_E$. We may think of $H(x, hD_x; h)$ as Schrödinger operator with Stark effect, with $M = \mathbb{R}^n$, or $H(x, hD_x; h)$ as the geodesic flow on an axially symmetric manifold $M$, extending Poincaré example of Lagrangian systems with 2 degree of freedom. By semi-hyperbolic, we mean that the linearized Poincaré map $P_E$ associated with $\gamma_E$ has at least one eigenvalue of modulus greater (or less) than 1, and by non-degenerate that 1 is not an eigenvalue. We look for semi-excited resonances near $E$, i.e. with imaginary part of magnitude $h^s$, with $0 < s < 1$. It is known that these resonances are given by the zeroes of a determinant $\zeta(z, h) = \det(I - M(z, h))$ where $M(z, h)$ is the monodromy operator. We make here this result more precise, in providing a first order asymptotics for Bohr-Sommerfeld quantization rule associated with $\gamma_E$ in terms of the (real) longitudinal and (complex) transverse quantum numbers, including the action integral, the sub-principal 1-form and Gelfand-Lidskii (or Cohnley–Zehnder) index. This index enters only when $P_E$ has some eigenvalue of modulus 1. In the latter case, a theorem of Lewis and Birkhoff shows that an infinite number of periodic orbits cluster near $\gamma_E$, with periods approximately multiples of the primitive period of $\gamma_E$. Extending some result by Sjöstrand and Zworski, we propose also a generalized Poisson formula expressing $\text{Tr} f(H/h)$ microlocally near $\gamma_E$, as a truncated series (depending on $h$) of integrals of the form $\text{Tr} \int f(z/h)M(z, h)^K \frac{dK}{K!}M(z, h)dz$. 

References
How to hear the corners of a drum

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Analytically computing the spectrum of the Laplacian is impossible for all but a handful of classical examples. Consequently, it can be tricky business to determine which geometric features are spectrally determined; such features are known as geometric spectral invariants. Weyl demonstrated in 1912 that the area of a planar domain is a geometric spectral invariant. In the 1950s, Pleijel proved that the perimeter is also a spectral invariant. Kac, and McKean & Singer independently proved in the 1960s that the Euler characteristic is a geometric spectral invariant for smoothly bounded domains. At the same time, Kac popularized the isospectral problem for planar domains in his article, “Can one hear the shape of a drum?” Colloquially, one says that one can “hear” spectral invariants. Hence the title of this talk in which we will show that the presence, or lack, of corners is spectrally determined. In the process, we will see how a certain “corner contribution” to the heat trace is obtained by explicitly calculating the Green’s functions for infinite sectors with Dirichlet, Neumann, Robin, and mixed boundary conditions. Moreover, using microlocal techniques, we will see that this corner contribution is universal. Finally, we will show how the results generalize to surfaces. This talk is based on current joint work with M. Nursultanov and D. Sher, and previous joint work with Z. Lu.

References


Dirichlet problem for mixed type equation with characteristic degeneration

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Consider the elliptic-hyperbolic equation

$$Lu = u_{xx} + (\text{sgn} \ y) |y|^nu_{yy} + a|y|^{n-1}u_y - b^2u = 0$$  

in the rectangular domain $D = \{(x, y) | 0 < x < l, -\alpha < y < \beta\}$, where $l > 0, \alpha > 0, \beta > 0, 0 < a < 1, n < 1$ are given positive real numbers.
The Dirichlet problem. Find a function \( u(x, y) \) satisfying the following conditions:

\[
\begin{align*}
    u(x, y) &\in C^2(D_+ \cup D_-) \cap C(D); \\
    \lim_{y \to 0^+} y^a u_y(x, y) &= \lim_{y \to 0^-} (-y)^a u_y(x, y); \\
    Lu(x, y) &\equiv 0, \quad (x, y) \in D_+ \cup D_-; \\
    u(x, \beta) &= f(x), \quad u(x, -\alpha) = g(x), \quad 0 \leq x \leq l, \\
    u(0, y) &= 0, \quad -\alpha \leq y \leq \beta; \\
    u(l, y) &= u(0, y), \quad -\alpha \leq y \leq \beta,
\end{align*}
\]

where \( D_+ = D \cap \{ y < 0 \}, \quad D_- = D \cap \{ y > 0 \}, \quad f(x) \) and \( g(x) \) are given sufficiently smooth functions, \( f(0) = f(l) = 0 \), and \( g(0) = g(l) = 0 \).

In the present paper based on works [1], [2], necessary and sufficient conditions of uniqueness of a solution to problem (2)–(7) for Eq. (1) in a rectangular domain are more clearly identified. A solution is constructed in the form of a series with respect to the system of eigenfunctions of the corresponding one-dimensional spectral problem. When substantiating the convergence of the series, a problem of small denominators with respect to \( 2\alpha^q/l, q = (2-n)/2 \), arises. In this connection, we establish estimates about separation of a small denominator from zero with the corresponding asymptotics for rational and irrational values of the number \( 2\alpha^q/l \), which permit one to establish the convergence of the constructed series in the class of regular solutions (2) and (3).

This work was supported by RFBR grant No. 18-31-00111.

**References**


Steady-state motion of charged dust particles under gravitational forces and forces of inertia

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We study the time evolution of an ensemble of charged dust particles. They evolve according to the Vlasov–Poisson equations. The problem we have to solve here consists in adding to the electric field produced by the charged particles the gravitational forces and the forces of inertia. This setting emerges from the problem of elusive Kordylewski clouds - dust particles in vicinity of the Lagrange libration points of the Earth-Moon system [1]. The mathematical approach is based on the reduction of the stationary Vlasov equation by means of energy substitution, and following analysis of the system of non-linear elliptic equations [2].

References

Analytic and algebraic indices of elliptic operators associated with quantized canonical transformations

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We consider elliptic operators associated with discrete groups of quantized canonical transformations as defined in [1] In order to be able to apply results from algebraic index theory, we define the localized algebraic index of the complete symbol of an elliptic operator. With the help of a calculus of semiclassical quantized canonical transformations, a version of Egorov’s theorem and a theorem on trace asymptotics for semiclassical Fourier integral operators we show that the localized analytic index
and the localized algebraic index coincide. As a corollary, we express the Fredholm index in terms of the algebraic index for a wide class of groups, in particular, for finite extensions of Abelian groups.

References

Some applications of asymptotic solutions of Boussinesq type equations for the approximation of 2011 tsunami mareograms from DART stations

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In article [1] an asymptotic solution of the linearized Boussinesq equation (from the theory of gravitational water waves) with a localized initial condition was obtained. In article [2], the accuracy of this asymptotic solution was estimated. We apply these formulas to calculate the waves of the Japanese tsunami in 2011 and compare them with the real records of the deep–water stations DART. This work was supported by the grant of the RNF 16-11-10282.

References

Quantum Hall effect and noncommutative geometry

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We present an interpretation of quantum Hall effect in terms of noncommutative geometry. The essence of this effect is that for very low temperatures in the graph of dependence of Hall conductivity on filling factor there appear horizontal plateau, corresponding to integer (in appropriate units) values of the conductivity. In other
words, the Hall conductivity for low temperatures is “quantized.” Using noncommu-
tative geometry ideas, it is possible to construct a cyclic cocycle which integrality
implies the quantization of Hall conductivity.

Asymptotics
of the solution explicit difference scheme
for the wave equation with localized initial data

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We consider the Cauchy problem for the wave equation with localized data

\[ u_{tt} = c^2(x)u_{xx}, \ u|_{t=0} = V(x/\mu), \ u_t|_{t=0} = 0, \ x \in \mathbb{R}, \]

where \( V(y) \) is a smooth fast-decaying function with derivations and the parameter \( \mu \ll 1 \) is the parameter of the localization, function \( c(x) \) is smooth and bounded. The asymptotic solution of this problem can be constructed with the help of the Maslov’s canonical operator.

On the other hand the solution of this problem can be obtained via the numerical methods due to solving the difference scheme. The difference scheme can be written as a pseudo-differential operator due to shifting operator \( T u = e^{h \pi x} u \), where \( h \ll 1 \) is a step of discretization in the scheme.

Thus the difference scheme can be also studied with the help of the Maslov’s theory and the asymptotic solution of the difference scheme can be studied. We investigate the asymptotic solution of the explicit difference scheme. We compare the numerical solution and asymptotic solution of the difference scheme. Even the explicit scheme is unstable one can provide interesting results of such comparison depending the ratio between the step of the difference scheme \( h \) and parameter of localization \( \mu \).

This work was supported by the Russian Science Foundation (project 16-11-10282).

References
Laplacians and wave equations on two-dimensional polyhedra

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We study Laplacians and wave equations on 2D polyhedra. Laplacians are defined by setting self-adjoint boundary condition in vertices. We study kernels of these operators and prove that they can be expressed in terms of the Mittag-Leffler problem on the corresponding Riemann surface. We also obtain trace formulas for the heat kernel. For the wave equation, we describe certain exact formulas for the solution of the Cauchy problem in terms of special integral transform as well as asymptotic support of localized asymptotic solutions. We also discuss statistics of localized waves.

Sobolev multipliers and their applications to differential and integral operators

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The talk is a survey of the theory of pointwise multipliers in spaces of differentiable functions which was developed together with Vladimir Mazya and summarised in our book “Theory of Sobolev multipliers and their applications to differential and integral operators”, Springer, 2009. In particular, I shall discuss explicit characterizations of multipliers in pairs of Sobolev spaces, Bessel potential spaces, and Besov spaces. Sharp results on essential norms and compactness properties of the multipliers, as well as trace, extension and composition theorems for the multipliers will be presented. I shall also give various applications of our theory to differential and integral operators, addressing such topics as regularity of the boundary for elliptic boundary value problems and continuity of general differential operators in pairs of Sobolev spaces.

Bruhat order and the symmetric Toda flow on real Lie groups

Georgy Sharygin
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My talk will describe a next step of our joint investigation with Yu. Chernyakov and A. Sorin, see [1, 2, 3]. It is dedicated to the generalizations of the theory of Toda flow, relating it with the theory of Lie groups and algebras. Namely, I will describe the way this system can be defined for arbitrary real form of a semisimple Lie algebra, so that the role of orthogonal group will fall on the maximal compact subgroup $K$ of the corresponding real group $G$. 
The dynamical system that I will talk about “lives” on the real flag space \(G/K\) of the real group; it has been studied in papers \([4, 5, 6]\) and many others. In particular, it was shown there that it is equal to the gradient flow of a Morse function (in generic case). Its constant points correspond to the elements of Weil group \(W\) of \(G\) and we shall show that (for normal real forms) trajectories of the system are “governed” by the Bruhat order on \(W\); an interesting consequence of this fact is that the real Bruhat cells intersect (transversally) the dual cells if and only if the corresponding elements in \(W\) are comparable in Bruhat order. So far this fact has been established only in complex case; and in the case \(G = \text{SL}_n(\mathbb{R})\) we used this property (which can be proved by the virtue of matrix representation) to describe the trajectories of Toda flow. There are many evidences that a similar property should hold for non-split real forms as well, although the size of the intersections should also depend on the dimensions of eigenspaces. Besides this, if time permits, I will also describe the invariants of this system, which will depend on the structure of the representations of \(G\).

References


On perturbations of self-adjoint and normal operators: analytical aspects

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We shall talk about spectral properties of operators of the form \(A = T + B\), where \(B\) is a non-symmetric operator subordinated to a self-adjoint or normal operator \(T\). An operator \(B\) is said to be \(T - p\)-subordinated (\(0 \leq p < 1\)) if the domain of \(B\) contains the domain of \(T\) and

\[
\|Bx\| \leq b\|Tx\|^p \|x\|^{1-p} \quad \forall \ x \in \mathcal{D}(T) \subset \mathcal{D}(B), \quad b = \text{const}.
\]

There is a great number of results (in particular, by J. Birkhoff, T. Carleman, M.V. Keldysh, F. Brauer, S. Agmon, V.B. Lidskii, I. Ts. Golberg and M.G. Krein,
F. S. Markus, V. I. Matsaev, B.S. Mityagin) which say about completeness and basis property of the eigenfunctions and preserving of the eigenvalue asymptotics of self-adjoint operators under $p$-subordinated perturbations.

We introduce new concepts of local subordination and local subordination in the sense of quadratic forms and prove new theorems which involve new technique and can be applied for concrete problems in more general situation.

In the second part we will discuss problems on perturbations of a self-adjoint operator $T$ with continuous spectrum whose spectrum consists of infinitely many segments $[\sigma_k, \sigma_{k+1}]$ separated by gaps: $\text{dist}(\sigma_k, \sigma_{k+1}) \geq \text{const.}$

The work is supported by the Russian Science Foundation, grant No 17-11-01215.

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**Study of some general classes of estimators for estimating population mean in compromised imputation**

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In this paper, three generalized classes of estimators in compromised imputation have been suggested and their corresponding point estimators of the population mean are obtained. Their biases, mean square error (MSE) expressions and percentage relative efficiency (PRE) are obtained and their optimum estimators are compared with the sample mean, ratio and compromised method of imputation and necessary conditions are derived. Further, theoretical results have been verified through empirical studies with the help of some natural population data sets to compare their efficiencies.

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**Stationary and nonstationary solutions of mixed problems for Vlasov–Poisson equations**

A. L. Skubachevskii

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A mathematical model of high temperature rarefied plasma in thermonuclear fusion reactor is described by the Vlasov-Poisson equation. If considerable number of charged particles reach the wall of the vacuum chamber of the reactor, then either the reactor wall will be destroyed, or the high-temperature plasma will be cooled. We consider a two-component rarefied plasma with external magnetic field in infinite cylinder. To obtain plasma confinement an external magnetic field is used, i.e. we
are looking for solutions with supports, which are situated at some distance from the cylindrical boundary. It were constructed stationary solutions with this property. It is also proved that for sufficiently strong magnetic field and sufficiently small initial density distribution functions the Vlasov-Poisson system has a unique classical solution with support being at some distance from the boundary [1, 2].

The publication was prepared with the support of the “RUDN University Program 5-100” and the Russian Foundation for Basic Research, grant number 17-01-00401.

References

Locally bounded finally precontinuous finite-dimensional quasirepresentations of connected locally compact groups

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We describe the structure of finite-dimensional locally bounded finally precontinuous quasirepresentations of arbitrary connected locally compact groups.

References

Lower bounds of Lipschitz constants on foliations

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In this talk, we will discuss Llarull's theorem in the foliation case and a lower bound of the Lipschitz constant of the map $M \to S^n$ in the foliation case under the spin condition.

References
Approximate solution of fractional order partial differential equations and its coupled systems using operational matrices approach

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In this article, an accurate numerical method based on operational matrices of fractional order derivatives and integrals in the Caputo and Riemann-Liouville senses of two-parametric orthogonal shifted Jacobi polynomials is proposed for studying the approximate solutions of FOPDEs. The technique is extended herein to generalized classes of fractional order coupled systems having mixed partial derivatives terms. One salient aspect of this article is the development of a new operational matrix for mixed partial derivatives in the sense of Caputo. Furthermore, as a result of the comparative study, some results presented in the literature are extended and improved in the investigation herein.

References
Solution of the equations describing the plasma in a gas discharge

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The Protosfera project [1] is aimed at producing hot toroidal plasma around a centerpost plasma discharge. Toroidal magnetic field is produced by current driven by a DC voltage between electrodes placed at the edges of the centerpost discharge. This work presents a preliminary analysis of momentum balance in the centerpost plasma. Plasma density and magnetic field data indicate that a multifluid description of the plasma can be applied, but Larmor gyration, electron-ion collisions and plasma-neutrals collisions have to be kept into account as a whole. A set of fluid equations sufficiently general to provide a realistic description is presented. Solutions in simplified geometry are given, which provide insight into the dominant mechanisms. We found reasonable hypothesis in order to deal with the complicated non-linear mechanisms acting in the system like electron-ion collisions and plasma-neutrals collisions.

References

Asymptotics of the solution of the Klein–Gordon equation with localized initial conditions

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The talk will be devoted to the Klein–Gordon equation with localized initial conditions. We will give the asymptotics of the solution and simplify the asymptotic formulas for various relations between the mass parameter and the width of the initial condition. It will be shown that for some simple initial condition the asymptotics of the solution is expressed in terms of the Hankel function of the first kind. This work was supported by the grant of the RNF 16-11-10282.
Pseudo-differential operators, equations, and elliptic boundary value problems

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We study Fredholm properties for special types of pseudo-differential operators which are constructed by their local representatives, so-called operators of local type \[1, 4\].

Let \(M\) be a smooth \(m\)-dimensional manifold with boundary \(\partial M\) which has some singularities \(M_k, k = 0, 1, \ldots, m - 2\). There are some smooth compact sub-manifolds \(M_k\) of dimension \(0 \leq k \leq m - 2\) on the boundary \(\partial M\) of the manifold \(M\) which are singularities of the boundary. These singularities are described by local representatives of the operator \(A\) in a point \(x \in M\) in the chart \(U \ni x\) in the following way

\[
 u(x) \mapsto \int_D \int_{\mathbb{R}^m} e^{i\xi \cdot (x-y)} A(\psi^{-1}(x), \xi) u(y) d\xi dy, \quad x \in D,
\]

where \(A(x, \xi)\) is an elliptic symbol defined on a special bundle over the manifold \(M\), \(\psi: U \to D\) is a diffeomorphism, and the canonical domain \(D\) has a special form depending on a location of the point \(x\) on manifold \(M\).

Such an operator \(A\) will be considered in Sobolev–Slobodetskii spaces \(H^s(M)\), and local variants of such spaces will be spaces \(H^s(D)\) where \(D\) is one of the following canonical domains \(\mathbb{R}^m, \mathbb{R}^m_+ = \{x \in \mathbb{R}^m : x = (x', x_m), x_m > 0\}, W^k = \mathbb{R}^k \times C^{m-k}\), and \(C^{m-k}\) is a convex cone in \(\mathbb{R}^{m-k}\) non-including a whole straight line. For each point \(x \in \partial M\) we define and assume the existence of special factorization for the elliptic symbol \(A(x, \xi)\) with index \(\alpha_k(x), k = 0, 1, \ldots, m - 1 \ [3, 5]\), and \(\alpha_{m-1}(x)\) is an index of factorization in Eskin’s sense \[2\] for smoothness points \(x \in \partial M\).

**Theorem.** If

\[
 |\alpha_k(x) - s| < 1/2, \quad \forall x \in M_k, \quad k = 0, 1, \ldots, m - 1,
\]

then the operator \(A: H^s(M) \to H^{s-\alpha}(M)\) is a Fredholm operator.

**Remark.** If an ellipticity property does not hold on sub-manifolds \(M_k\) then one can consider a modification of the operator \(A [3, 5]\).

This work was supported by the State contract of the Russian Ministry of Education and Science (contract No 1.7311.2017/8.9).

**References**


Consider a part of the plane bounded by arcs of confocal quadrics. Such billiard is integrable. More precisely, the straight lines containing the segments of the polygonal billiard trajectory are tangent to a certain quadric (ellipse or hyperbola). This quadric belongs to the same class of confocal quadrics as the quadrics whose arcs form the boundary of the domain (billiard). Such billiard we call elementary billiard.

Let $B_0$ be an elementary billiard which has an empty intersection with the focal line and bounded by two arcs of hyperbolas (convex and nonconvex) and two arcs of ellipses. We glue a number of such billiards into a two-dimensional torus $\Delta T(B_0)$. Distinguish on it a convex parallel and a meridian — a union of convex gluing edges lying on elliptic and hyperbolic segments, respectively. These two parallels and two meridians break the $\Delta T(B_0)$ torus into four leaf regions. We denote them by $\alpha$, $\beta$, $\gamma$, and $\delta$. At the same time, the pairs of sheets $\alpha$, $\gamma$ and $\beta$, $\delta$ are glued along hyperbolic borders, denoted by $a$ (convex) and $b$ (nonconvex). Pairs of sheets $\alpha$, $\beta$ and $\gamma$, $\delta$ are glued along elliptic borders, denoted by $c$ (convex) and $d$ (non-convex).

Let $n$ and $k$ be mutually simple positive integers, and $k < n$. Let $\sigma$ denote the permutation $(1 \ 2 \ \ldots \ n)$. Consider $n$ copies of billiards $\Delta T(B_0)$. We number the billiards with numbers from 1 to $n$ and continue the numbering on their sheets $\alpha$, $\beta$, $\gamma$, $\delta$. On the union of these billiards, we define the following motion. On the convex elliptic boundary $a$, the material point, when moving along the sheet $\alpha_i$ (corresponding to $\beta_i$), after impact hits the sheet $\gamma_{\sigma(i)}$ (corresponding to $\delta_{\sigma(i)}$), and when moving on the sheet $\gamma_i$ (corresponding to $\delta_i$), after the impact, moves to the sheet $\alpha_{\sigma^{-1}(i)}$ (corresponding to $\beta_{\sigma^{-1}(i)}$). On the convex hyperbolic boundary of $c$, the material point when moving along the sheet $\alpha_i$ (respectively $\gamma_i$) after impact goes to the sheet $\beta_{\alpha^{s_{\sigma(i)}}}$ (respectively $\delta_{\alpha^{s_{\sigma(i)}}}$), and when moving on the sheet $\beta_i$ (respectively $\delta_i$) after the impact, it moves to the sheet $\alpha_{\sigma^{-k}(i)}$ (respectively $\gamma_{\sigma^{-k}(i)}$). On nonconvex boundaries $b$ and $c$, the motion is defined without changing billiard $\Delta T(B_0)$. We denote this billiard (billiard book) by $T(\Delta(B_0), n, k)$.

Theorem. The Fomenko–Zieshang invariant classifying the Liouville foliation of the $Q^3$ isoenergy surface for an integrable topological billiard $\Delta T(B_0)$ has the form shown in the figure 1a), topological billiard books $\Delta T((B_0), n, k)$ has the form shown in the figure 1b).

This work was supported by the program “Leading Scientific Schools” (grant no. NSh-6399.2018.1).

References
The resolvent expansion of geometric operators on stratified spaces

Boris Vertman
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In this overview talk I will present a “direct” approach to the resolvent expansion of Laplace type operators on stratified spaces with iterated cone edge singularities. The paradigm is that on the model singularity one has a second order Sturm-Liouville operator with operator valued coefficients. This point of view has the advantage of being very elementary, microlocal analysis is only needed in the smooth interior. This is an joint work with Luiz Hartmann and Matthias Lesch.

Asymptotic of tunneling for Schrödinger equation with hyperbolic frequency resonance

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We consider the spectral problem for a two-dimensional Schrödinger operator $\hat{H}$ with hyperbolic resonance. Let a Wick symbol $\hat{H} = H(\bar{z}^*|\bar{z})$ be given by a sum of homogeneous functions in $(\bar{z}, z)$-coordinates

$$H = \frac{1}{\hbar}(H_0 + H_1 + H_2 + \ldots), \quad \deg H_j = j + 2, \quad N \geq 2,$$

Figure 1: Fomenko–Zieschang invariants of a series of topological billiards $\Delta T(B_0)$ (a) and topological billiard books (b).

where the harmonic summand $H_0$ is just the hyperbolic oscillator

$$H_0(z) = \omega_+ z_+ z_+ - \omega_- z_- z_-.$$ 

Under assumption that normal frequencies $\omega_+ > 0$ of the harmonic part $H_0$ are in resonance $\omega_+ = \omega_-$, we study the spectral problem for the operator $\hat{H}$ in the semiclassical approximation $\hbar \to 0$. The resonance leads to the appearance of a non-Lie algebra of symmetries (operators commuting with $\hat{H}_0$) with polynomial commutation relations.

Applying the operator averaging to the harmonic term $\hat{H}_1 + \hat{H}_2 + \ldots$, we obtain the effective operator $\hat{E}$ defined on irreducible representations of the symmetry algebra. We show that a bounded eigenstate of $\hat{E}$ can be localized in different regions of the classical phase space. In this case, we obtain an asymptotic for the corresponding tunnel energy splitting. This energy splitting is similar to the well-known tunnel splitting that appears in case of one-dimensional Schrödinger operator $\hat{H} = \hat{p}^2 + V(x)$ with a double-well potential $V(x)$. We also express the asymptotic of splitting in terms of the tunneling action and complex instantiations.

The details can be found in [1]. This work was supported by the Program for Fundamental Research of Higher School of Economics.

References

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Fried conjecture for Morse–Smale flow

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The relation between the spectrum of the Laplacian and the dynamical flow on a closed Riemannian manifold is one of the central themes in differential geometry. Fried [3] conjectured a relation between the analytic torsion, which is an alternating product of regularized determinants of the Hodge Laplacians, and the Ruelle dynamical zeta function. We will formulate and show this conjecture for Morse-Smale flow. Our proof relies on the Cheeger-Müller [2, 4]/Bismut-Zhang [1] theorem. This is joint work with Shu Shen.

This work was supported by NSFC (No. 11771411).

References
Positive scalar curvature on foliations

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The classical Lichnerowicz theorem states that the Hirzebruch A-roof genus of a compact spin manifold vanishes if the underlying manifold carries a Riemannian metric of positive scalar curvature. We will describe various generalizations of this result to the case of foliations.

The Maslov index in symplectic Banach spaces

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We consider a curve of Fredholm pairs of Lagrangian subspaces in a fixed Banach space with continuously varying weak symplectic structures. Assuming vanishing index, we obtain intrinsically a continuously varying splitting of the total Banach space into pairs of symplectic subspaces. Using such decompositions we define the Maslov index of the curve by symplectic reduction to the classical finite-dimensional case. We prove the transitivity of repeated symplectic reductions and obtain the invariance of the Maslov index under symplectic reduction, while recovering all the standard properties of the Maslov index. As an application, we consider curves of elliptic operators which have varying principal symbol, varying maximal domain and are not necessarily of Dirac type. For this class of operator curves, we derive a desuspension spectral flow formula for varying well-posed boundary conditions on manifolds with boundary and obtain the splitting formula of the spectral flow on partitioned manifolds.

References
Научное издание

МЕЖДУНАРОДНАЯ КОНФЕРЕНЦИЯ
ПО ДИФФЕРЕНЦИАЛЬНЫМ УРАВНЕНИЯМ
С ЧАСТНЫМИ ПРОИЗВОДНЫМИ
И ПРИЛОЖЕНИЯМ,
ПОСВЯЩЁННАЯ ПАМЯТИ
ПРОФЕССОРА Б.Ю. СТЕРНИНА

Издание подготовлено в авторской редакции